Modelling of bulk superconductor magnetisation: a review

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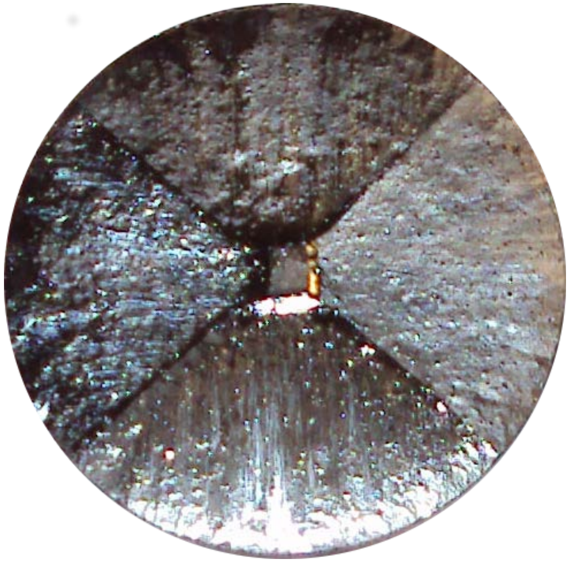
Presentation Outline

- **Overview of modelling bulk superconductor magnetisation**
- **Case studies:**
  - Multi-pulse, pulsed field magnetisation (PFM) of bulk high-temperature superconductors
  - Split coil PFM with an iron yoke
  - Field cooling (FC) magnetisation of iron-pnictide (Ba122) bulks

Bulk Superconductors

- Bulk superconducting materials can 'trap' large magnetic fields > 17 T
- Achieved by pinning penetrated magnetic field (quantised flux lines) → macroscopic electrical currents
- Magnetisation <u>increases</u> with sample volume



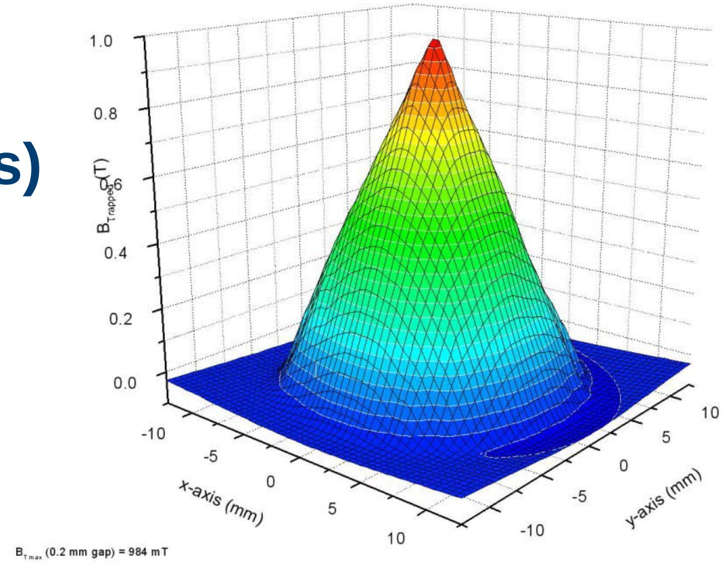
A large, single grain bulk superconductor

Bulk Superconductors

- **Bulk superconducting materials can ‘trap’ large magnetic fields > 17 T**
- **Achieved by pinning penetrated magnetic field (quantised flux lines) → macroscopic electrical currents**
- **Magnetisation <u>increases</u> with sample volume**
- **Trapped field given by**

$$B_{trap} = k\,\mu_0\,J_c\,R$$

where $$k = \frac{t_B}{2R} \cdot \ln\left(\frac{R + \sqrt{R^2 + {t_B}^2}}{t_B} \right)$$

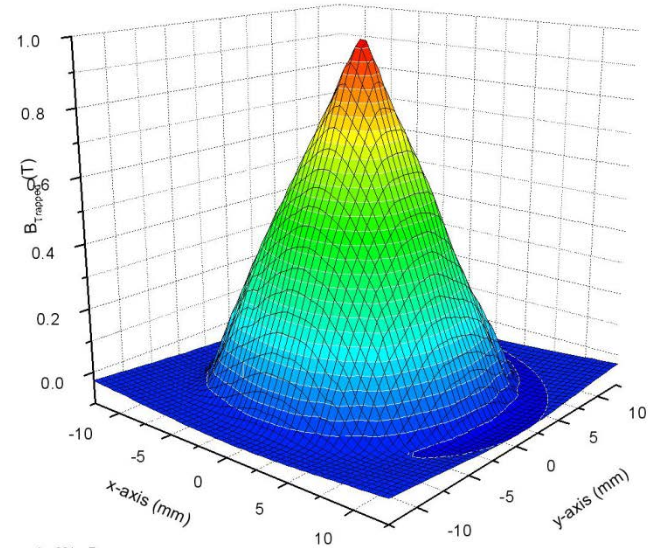


Typical trapped magnetic field profile of a bulk superconductor

Bulk Superconductors

$$B_{trap} = k\, \mu_0\, J_c\, R$$

where $$k = \frac{t_B}{2R} \cdot \ln\left(\frac{R + \sqrt{R^2 + {t_B}^2}}{t_B} \right)$$



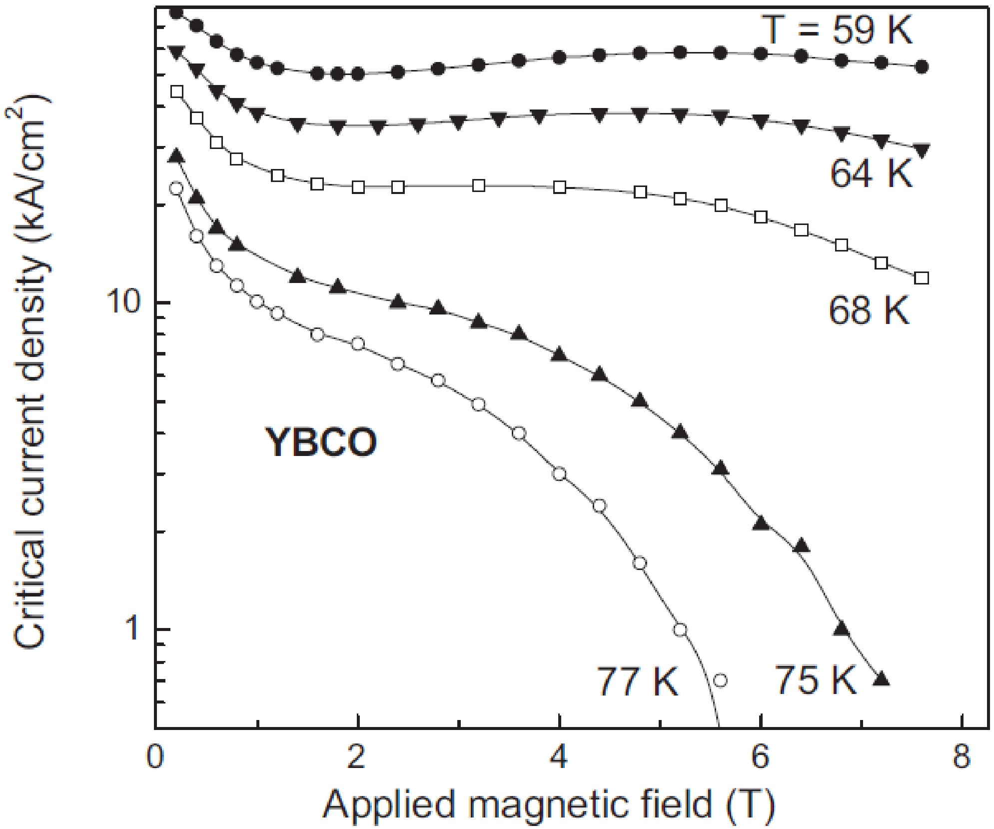
$B_{T\,max}$ (0.2 mm gap) = 984 mT

- **From Bean model (infinite slab) + Biot-Savart law**
- **Example of an <u>analytical</u> model**
  - Easier to deal with & faster
  - Rely on specific simplified geometries & simplified, homogeneous assumptions:
    - Constant / uniform J_c, no frequency / time dependence, etc.

Bulk Superconductors

$B_{trap} = k\,\mu_0 J_c R$

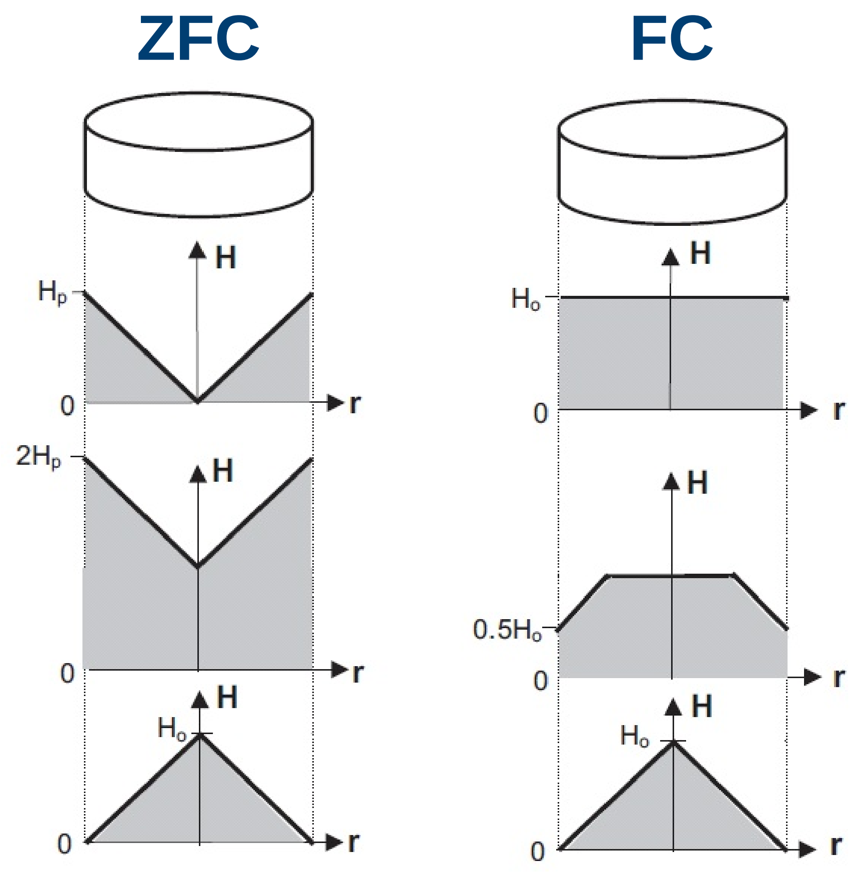
- **Candidate materials must be able to:**
  - Pin magnetic flux effectively
  - Carry large current density, J_c, over large length scales
  - Be insensitive to application of large magnetic fields, $J_c(B)$



Example field dependence of critical current density, $J_c(B)$, for bulk YBCO

Magnetisation of Bulk Superconductors

- **Three magnetisation techniques:**
  - Field Cooling (FC)
  - Zero Field Cooling (ZFC)
  - Pulsed Field Magnetisation (PFM)
- **To trap B_{trap}, need at least B_{trap} or higher**
  - FC and ZFC require large magnetising coils, long magnetizing times
  - Impractical for applications/devices → PFM



Numerical Modelling of Magnetisation

- **Numerical models can simulate practical & complex situations, playing a number of crucial roles:**
  - Simulate accurate magnetic field, current, temperature, mechanical stress distributions
  - Interpret experimental results & physical mechanisms of bulk superconductor magnetisation
  - Design & predict performance of magnetising fixtures & techniques
  - Design & predict performance of practical bulk superconductor-based devices

Numerical Modelling of Magnetisation

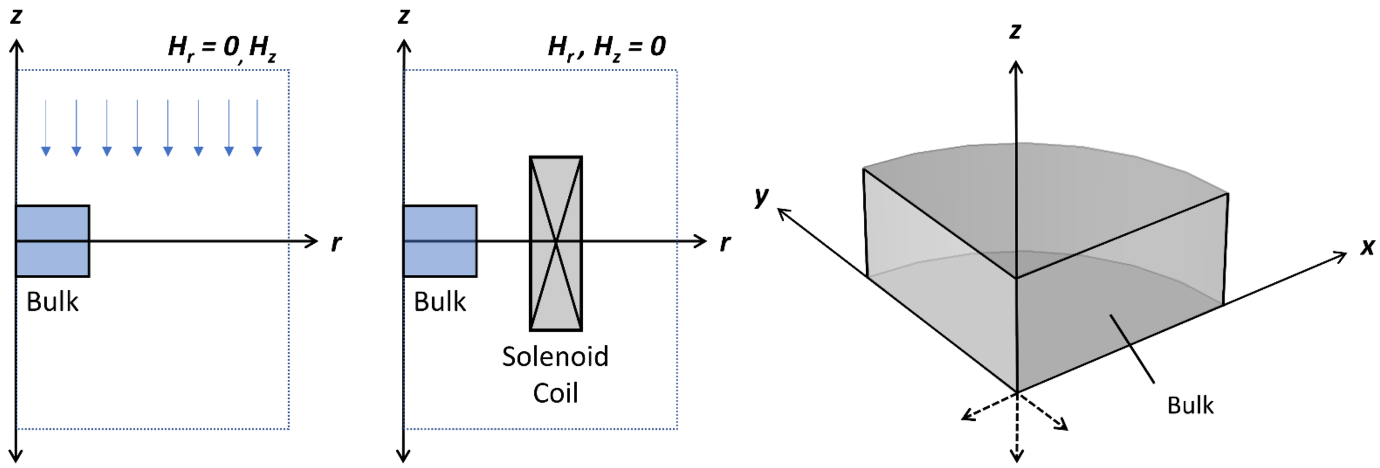
BULK GEOMETRY & MAGNETISATION FIXTURE

ELECTROMAGNETIC FORMULATION

THERMAL EQUATIONS & PROPERTIES

$J_c(B, T)$

E-*J* POWER LAW



- 2D axisymmetric generally sufficient for cylindrical bulks with a homogeneous J_c distribution
- 3D required for an inhomogeneous J_c distribution around the *ab*-plane; for non-symmetric shapes

Numerical Modelling of Magnetisation

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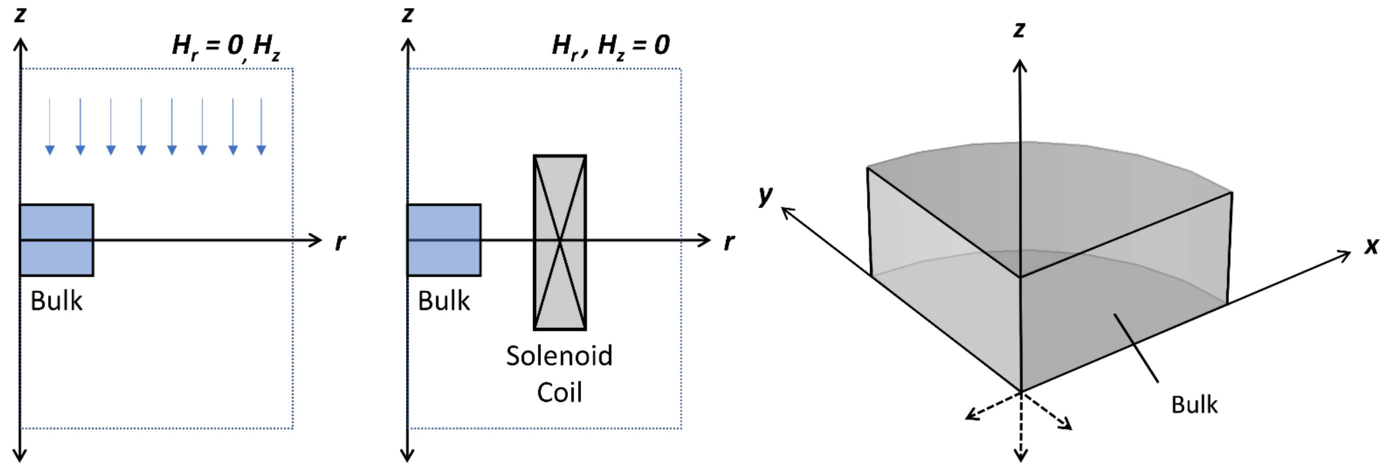

- Magnetising fixture: uniform boundary conditions or inserting a copper coil sub-domain
- Cooling: using a cold head + vacuum chamber or submersion in liquid cryogen

Numerical Modelling of Magnetisation

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E-*J* POWER LAW

Finite element method is commonly used & well developed (other techniques do exist)

Governing equations:
Maxwell's equations (***H*** formulation)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial(\mu_0 \mu_r \mathbf{H})}{\partial t}$$ Faraday's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$ Ampere's Law

Other formulations also exist (***A***-*V*, ***T***-*Ω, Campbell's equation*)

Numerical Modelling of Magnetisation

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$J_c(B, T)$

E-*J* POWER LAW

Thermal behaviour needs to be modelled when the bulk experiences a significant change in temperature

e.g., during PFM, modelling complete FC magnetisation process

Governing equations:

$$\rho \cdot C \frac{\mathrm{d}T}{\mathrm{d}t} = \nabla \cdot (k \nabla T) + Q$$

$$Q = E \cdot J$$

ρ = mass density, C = specific heat, κ = thermal conductivity, Q = heat source

Numerical Modelling of Magnetisation

BULK GEOMETRY & MAGNETISATION FIXTURE

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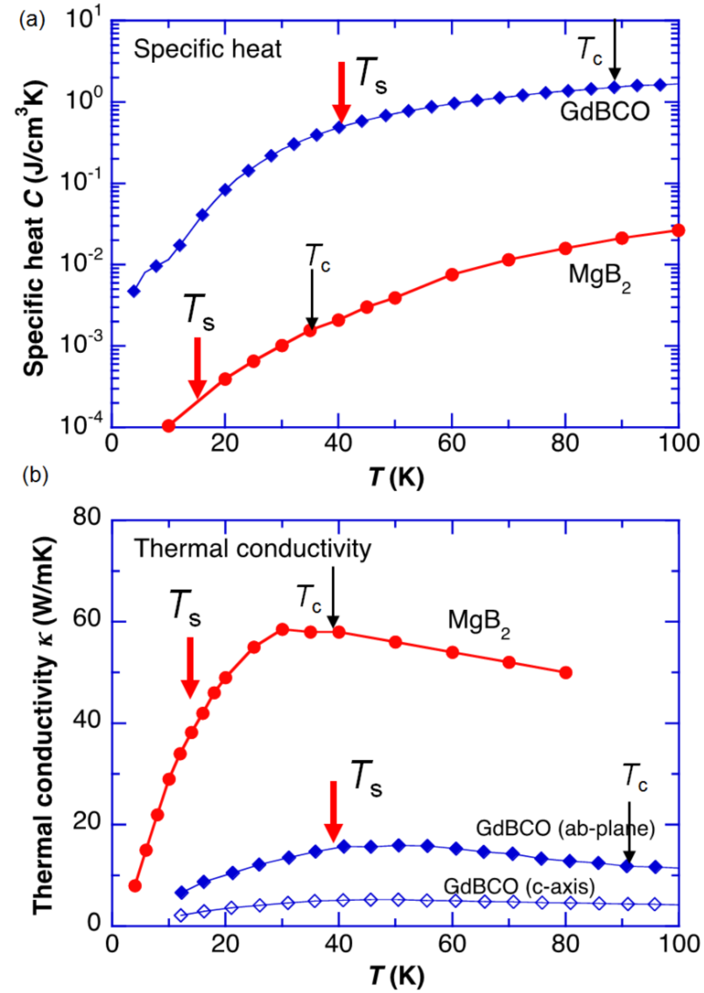
$J_c(B, T)$

E-*J* POWER LAW

Can use measured experimental data + fitting function or interpolation over a specific temperature range

Can choose constant parameters for *C*, *κ* for $T = T_{op}$ as a reasonable approximation



Numerical Modelling of Magnetisation

- BULK GEOMETRY & MAGNETISATION FIXTURE
- ELECTROMAGNETIC FORMULATION
- THERMAL EQUATIONS & PROPERTIES
- **$J_c(B, T)$**
- *E*-*J* POWER LAW

HTS materials

Kim-like model:

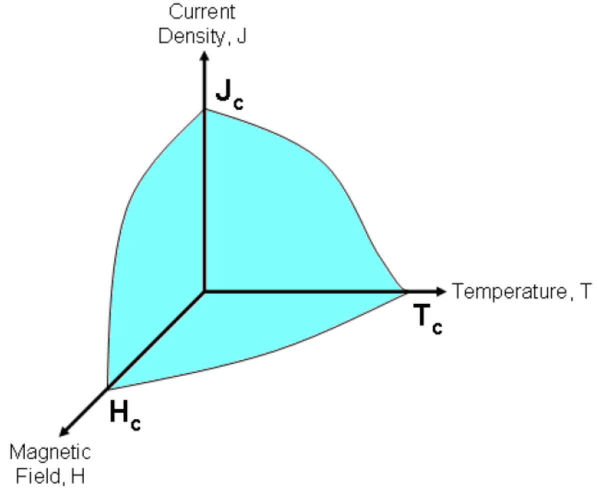
$$J_c = \frac{J_{c0}}{\left(1 + \frac{B}{B_0}\right)^{\alpha}}$$

Fishtail effect:

$$J_c(B) = J_{c1}\exp\left(-\frac{B}{B_L}\right) + J_{c2}\frac{B}{B_{max}}\exp\left[\frac{1}{y}\left(1-\left(\frac{B}{B_{max}}\right)^{y}\right)\right]$$

MgB_2 materials

$$J_c(B, T) = J_{c0}(T)\exp\left(-\frac{B}{B_0}\right)^{a}$$



$$J_{c0}(T) = \alpha\left[1 - \left(\frac{T}{T_c}\right)^{2}\right]^{1.5}$$

Numerical Modelling of Magnetisation

BULK GEOMETRY & MAGNETISATION FIXTURE

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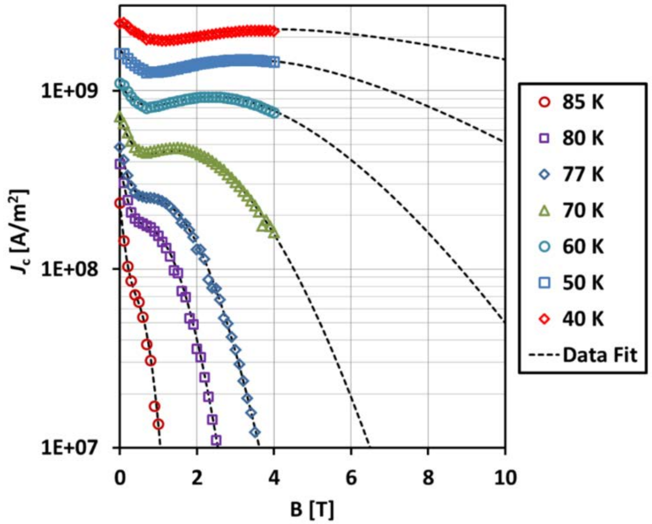
$J_c(B, T)$

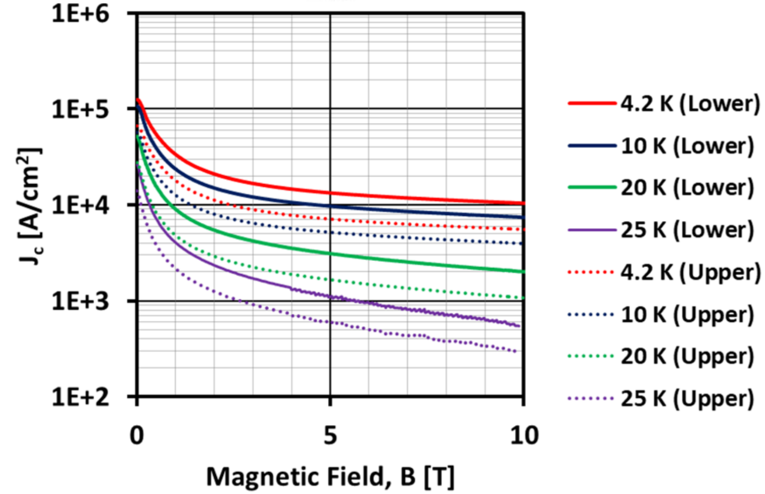
E-J POWER LAW

… or direct interpolation

$GdBa_2Cu_3O_{7-\delta}$ (15wt% Ag)



$Ba_{0.6}K_{0.4}Fe_2As_2$ (Ba122)



Numerical Modelling of Magnetisation

BULK GEOMETRY & MAGNETISATION FIXTURE

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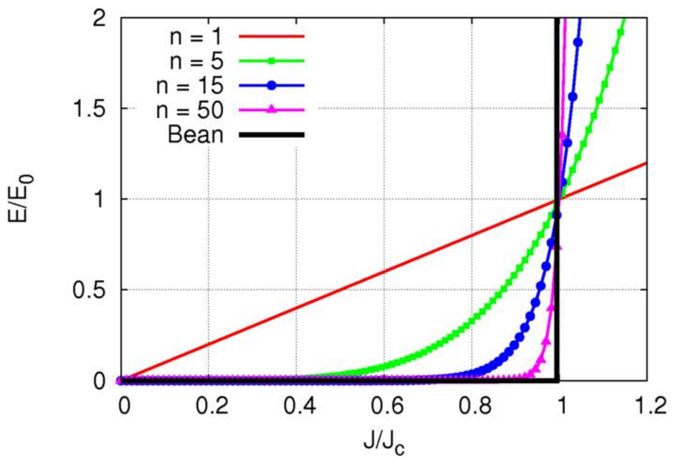
$J_c(B, T)$

***E*-*J* POWER LAW**

E-*J* power law

- Conventional materials → non-linear permeability, linear resistivity
- Superconductors → linear permeability (μ_0), non-linear resistivity
- Non-linearity is extreme: power law with $n > 20$



***I*-*V* curves for different *n* values**

$$\mathbf{E} = E_0 \left(\frac{J}{J_c}\right)^{n-1} \frac{\mathbf{J}}{J_c}$$

$$\boldsymbol{E} = \rho \boldsymbol{J}$$

$$\mathbf{E} = \boxed{E_0 \left(\frac{J}{J_c}\right)^{n-1} \frac{\mathbf{J}}{J_c}}$$

Case study #1:

Multi-pulse, pulsed field magnetisation (PFM) of bulk high-temperature superconductors

Pulsed Field Magnetisation

- **PFM technique: compact, mobile, relatively inexpensive**
- **Main issue: B_{trap} [PFM] < B_{trap} [FC], [ZFC]**
  - Temperature rise ΔT due to rapid movement of magnetic flux
- **Record PFM trapped field: 5.2 T @ 29 K**

  Top surface of 45 mm diameter Gd-Ba-Cu-O

  Fujishiro et al. *Physica C* 2006

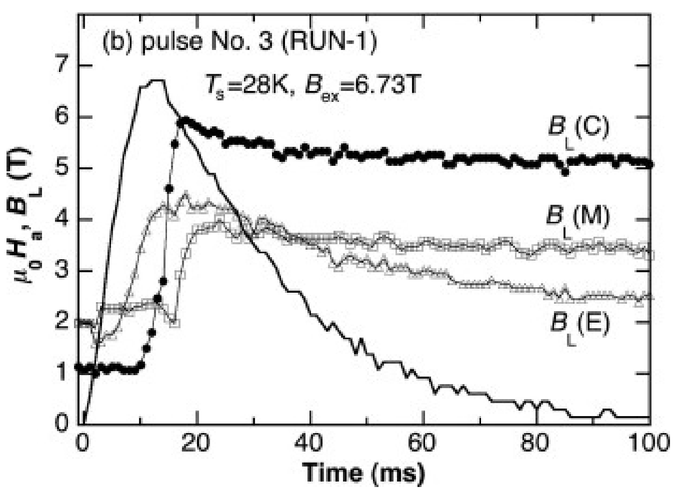
  - Record trapped field by FC: 17.6 T @ 26 K

    Centre of 2 x 25 mm diameter Gd-Ba-Cu-O

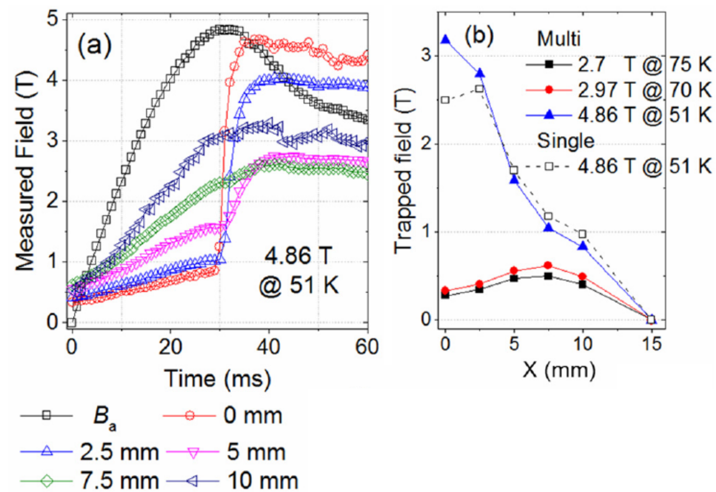
    Durrell et al. *Supercond. Sci. Technol.* 2014

Pulsed Field Magnetisation

- **Many considerations for PFM:**
  - Pulse magnitude, pulse duration, **<u>temperature(s)</u>**, **<u>number of pulses</u>**, type of magnetising coil(s), use of ferromagnetic materials
  - Dynamics of magnetic flux during PFM process
- **Multi-pulse PFM: effective in increasing trapped field/flux**



Fujishiro et al. *Physica C* 2006

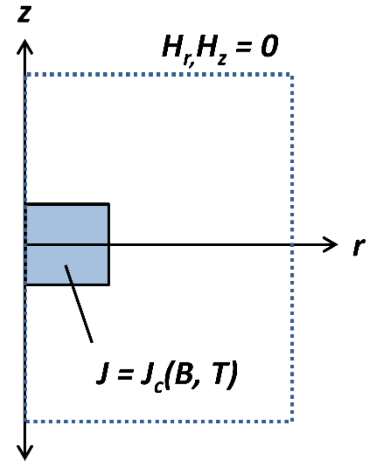


Zhou et al. *Appl. Phys. Lett.* 2017

Modelling Trapped Field Capability (FC/ZFC)

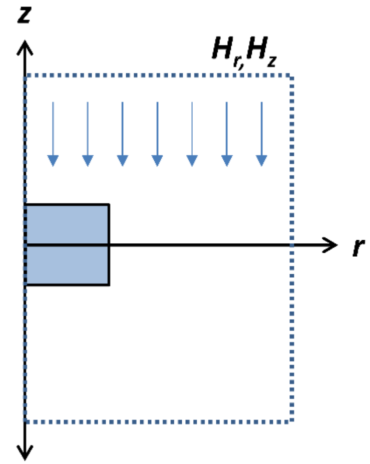
Model #1

- Stationary
- Comsol Multiphysics
  - 2D axisymmetric
  - AC/DC module
  - Magnetic Field (mf) interface
  - External Current Density node
- No flux creep
- Time taken: ~ 2-3 seconds



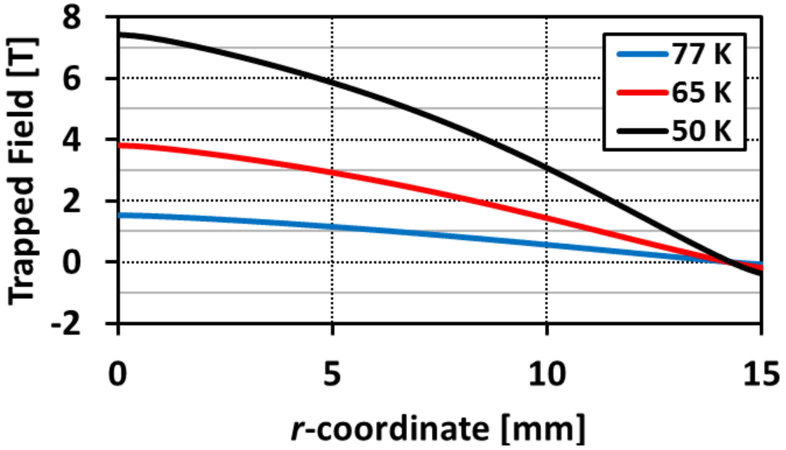
Model #2

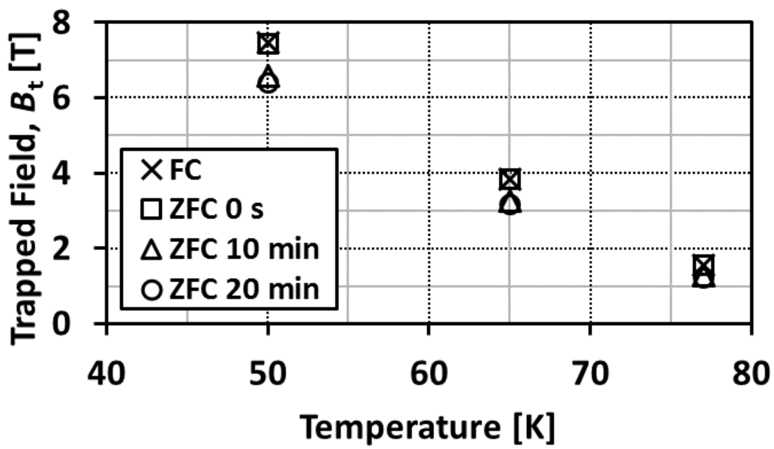
- Time-dependent
- Comsol Multiphysics
  - 2D axisymmetric
  - AC/DC module
  - Magnetic Field Formulation (mfh) interface
- *E*-*J* power law, $E \propto J^n$ (flux creep)
- Apply + remove background field
- Time taken: up to 1-2 hours



Modelling Trapped Field Capability

Results





$GdBa_2Cu_3O_{7-\delta}$ (15wt% Ag)

Magnetisation	Time	Trapped Field	
77 K			
FC	---	1.544 T	
ZFC [5 T]	t = 0 min	1.546 T	
	t = 10 min	1.263 T	
	t = 20 min	1.223 T	**79%**
65 K			
FC	---	3.826 T	
ZFC [10 T]	t = 0 min	3.827 T	
	t = 10 min	3.256 T	
	t = 20 min	3.158 T	**82.5%**
50 K			
FC	---	7.449 T	
ZFC [20 T]	t = 0 min	7.422 T	
	t = 10 min	6.577 T	
	t = 20 min	6.405 T	**86%**

PFM Modelling Framework

Electromagnetic properties modelled as Model #2 (ZFC); magnetising fixture assumed as solenoid coil:

$$I_{pulse}(t) = N \cdot I_0 \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right)$$

τ = 15 ms

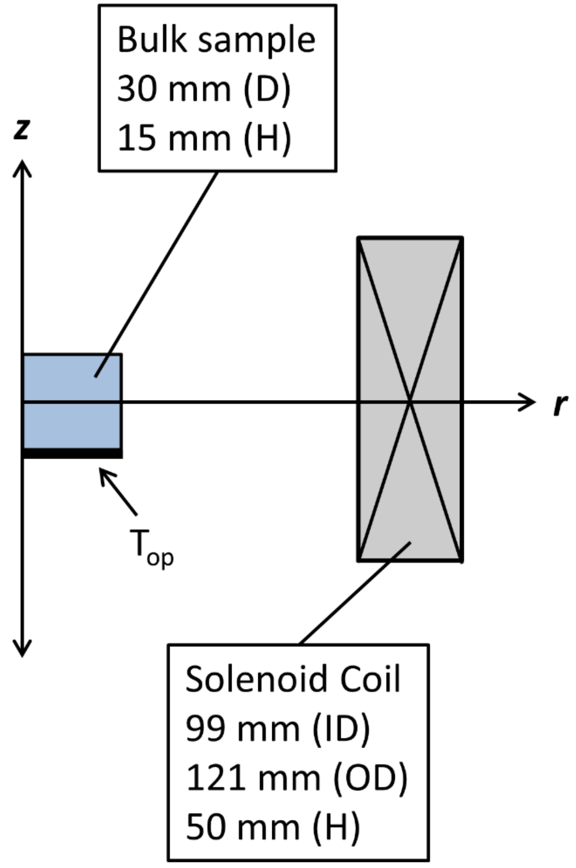
Thermal behaviour:

$$\rho \cdot C \frac{\mathrm{d}T}{\mathrm{d}t} = \nabla \cdot (k \nabla T) + Q$$

$$Q = E \cdot J$$

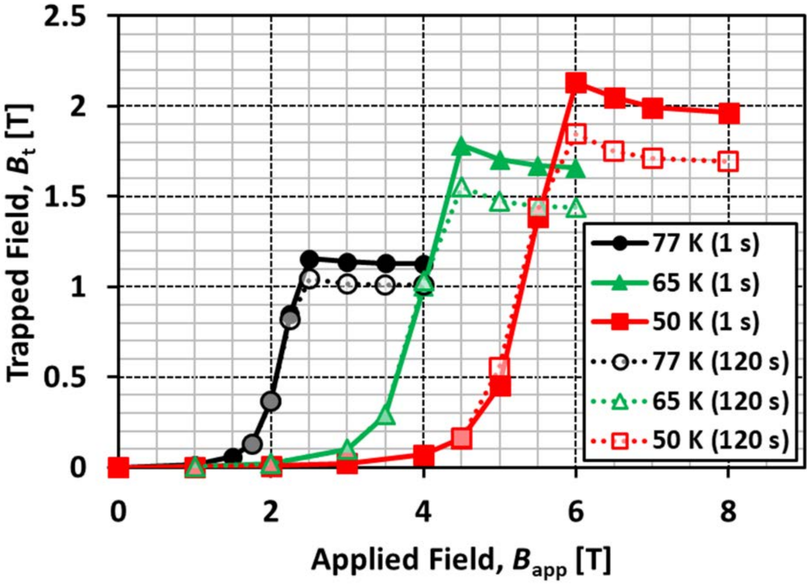
Heat source,
coupling with EM model + $J_c(B, T)$

ρ = mass density (bulk 5900 kg/m^3, indium 7310 kg/m^3)
C = specific heat (measured, temperature-dependent)
κ = thermal conductivity:
κ_{ab} = 20 W/(m·K), κ_c = 4 W/(m·K), κ_{indium} = 0.5 W/(m·K)

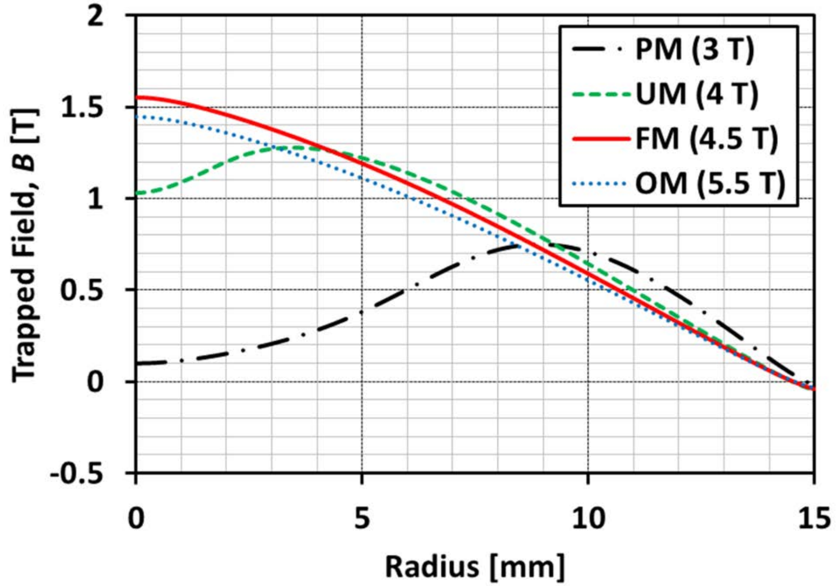


PFM Single Pulse Results

- *t* = 120 s → flux creep relaxation & cooling back to operating temp.
- Percentage of ZFC(*t* = 20 min):
  - 77 K 85%
  - 65 K 49%
  - 50 K 29%

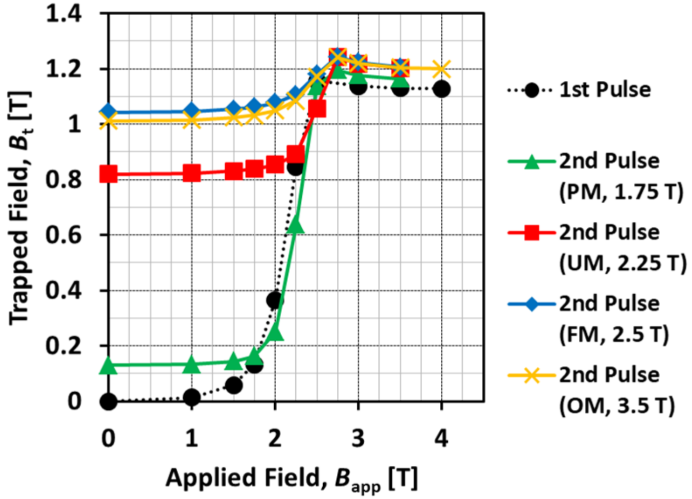


- Four specific cases as initial conditions for 2nd pulse:
  - Partially-magnetised (PM), so-called 'M-shaped' profile
  - Under-magnetised (UM)
  - Fully-magnetised (FM)
  - Over-magnetised (OM)

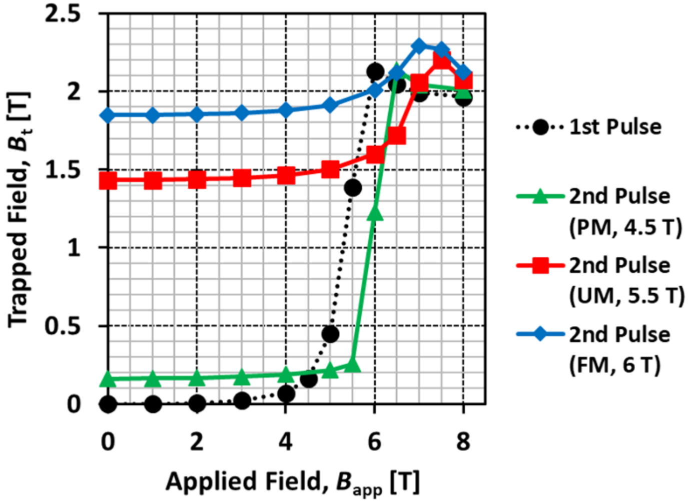


PFM 2nd Pulse Results

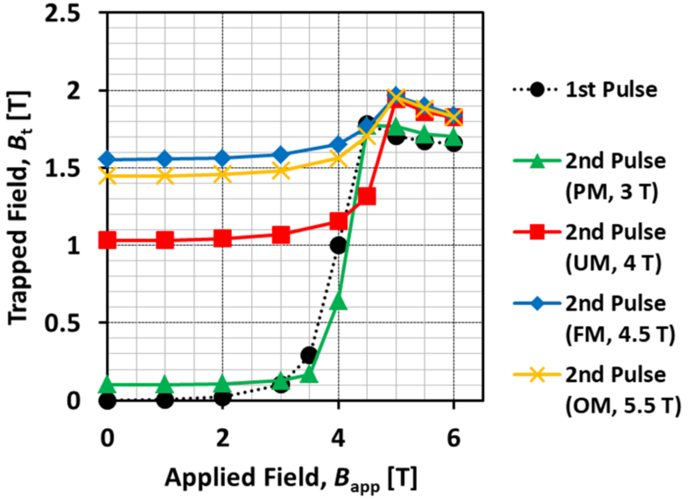
77 K



50 K



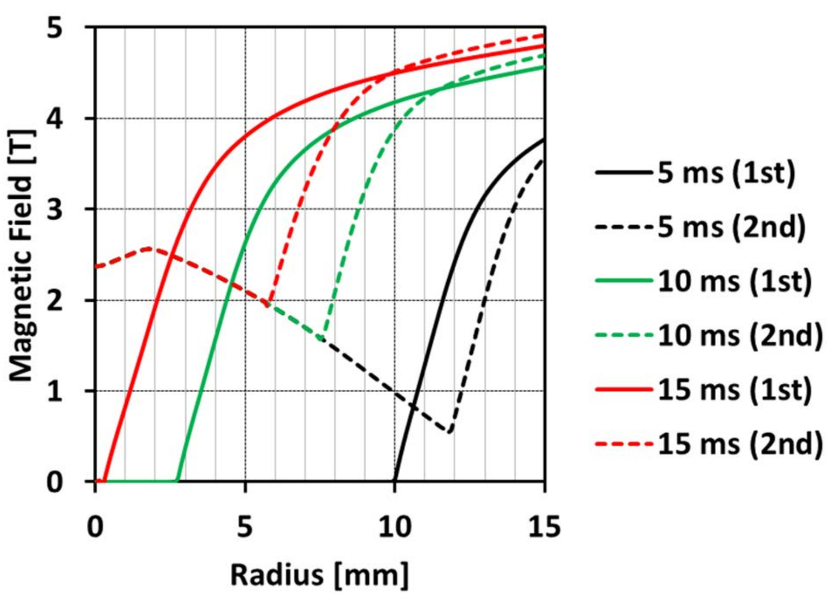
65 K

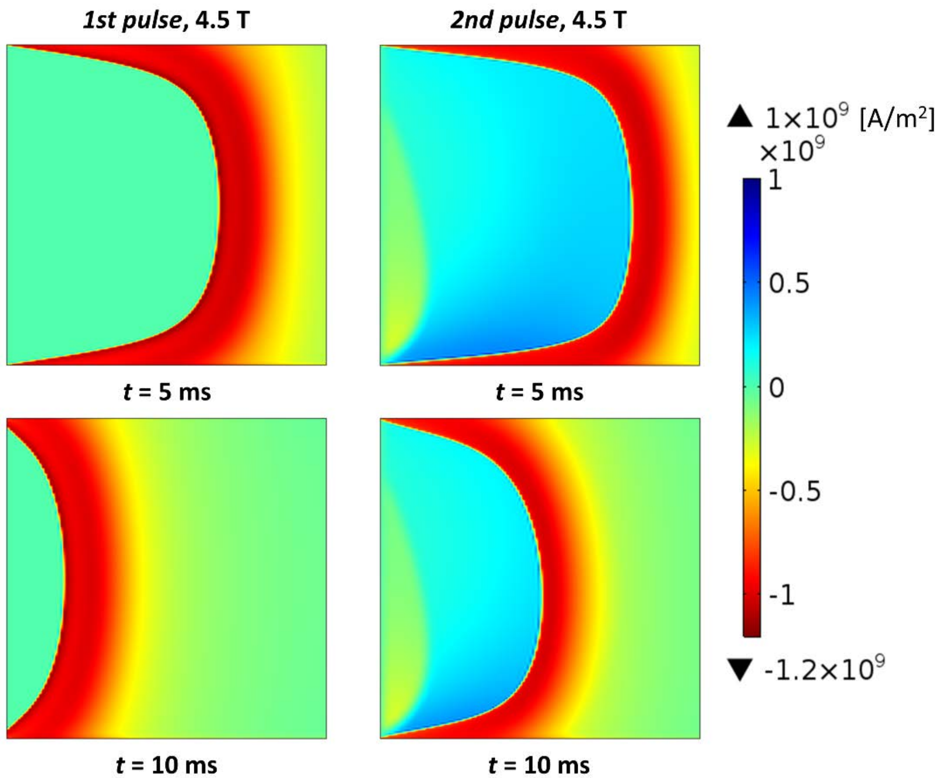


For all *T*, trapped field after 2nd pulse exhibits two particular characteristics:
1) Increased trapped field, B_t, when the bulk is fully magnetised; maximum value when the 1st pulse results in full magnetisation
2) Increased activation field: applied field, B_{app}, required to fully magnetise the sample

PFM 2nd Pulse: Magnetic Flux Penetration

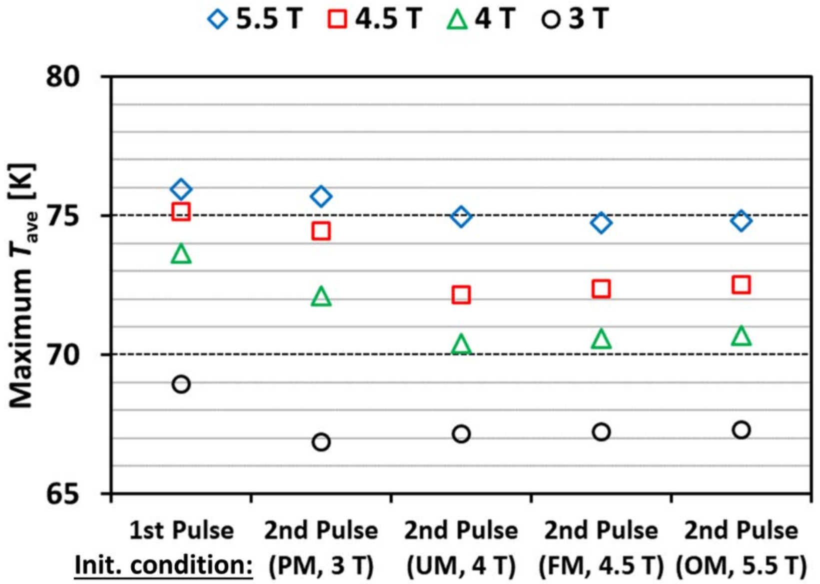
- Why does this occur?
  - More difficult for magnetic flux to penetrate the sample due to existing trapped field
  - Existing, induced supercurrent flows in opposite direction

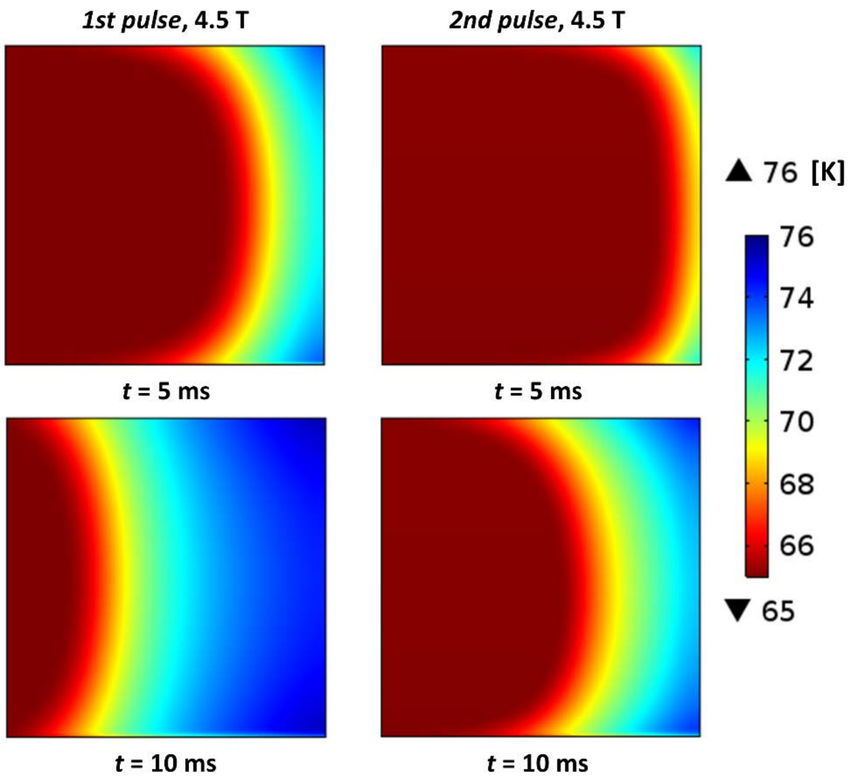




PFM 2nd Pulse: Thermal Behaviour

- Why does this occur?
  - Reduced dynamic movement of flux = lower temperature rise for equivalent next pulse
  - Can examine maximum average temperature, $T_{ave,max}$, during & after PFM:

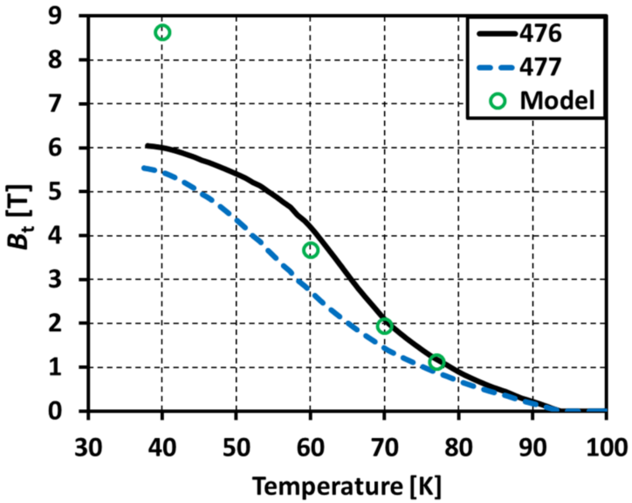


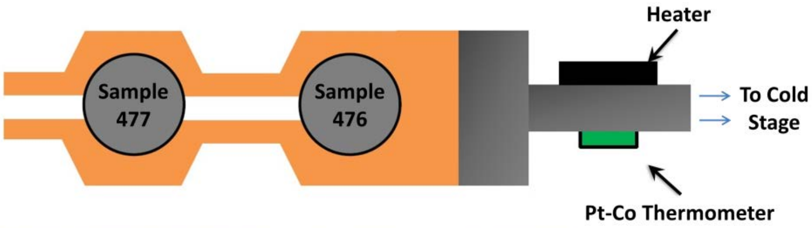


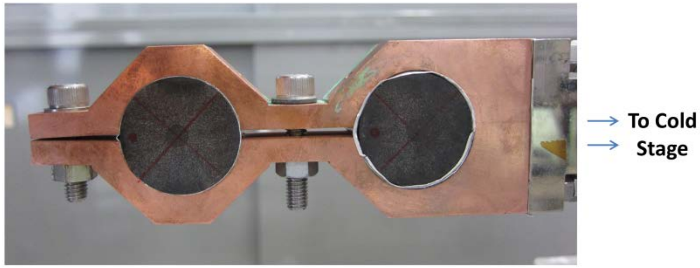
Case study #2:

Split coil PFM with an iron yoke

Split Coil PFM with an Iron Yoke

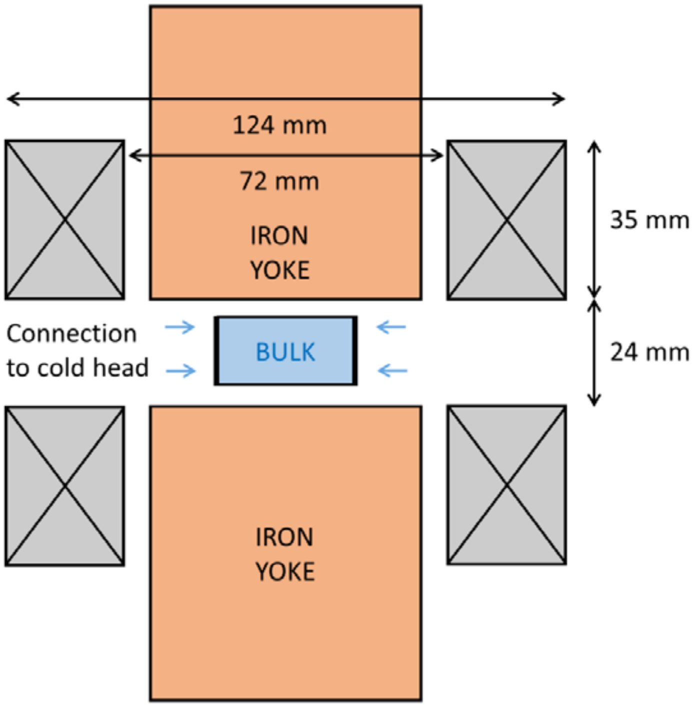






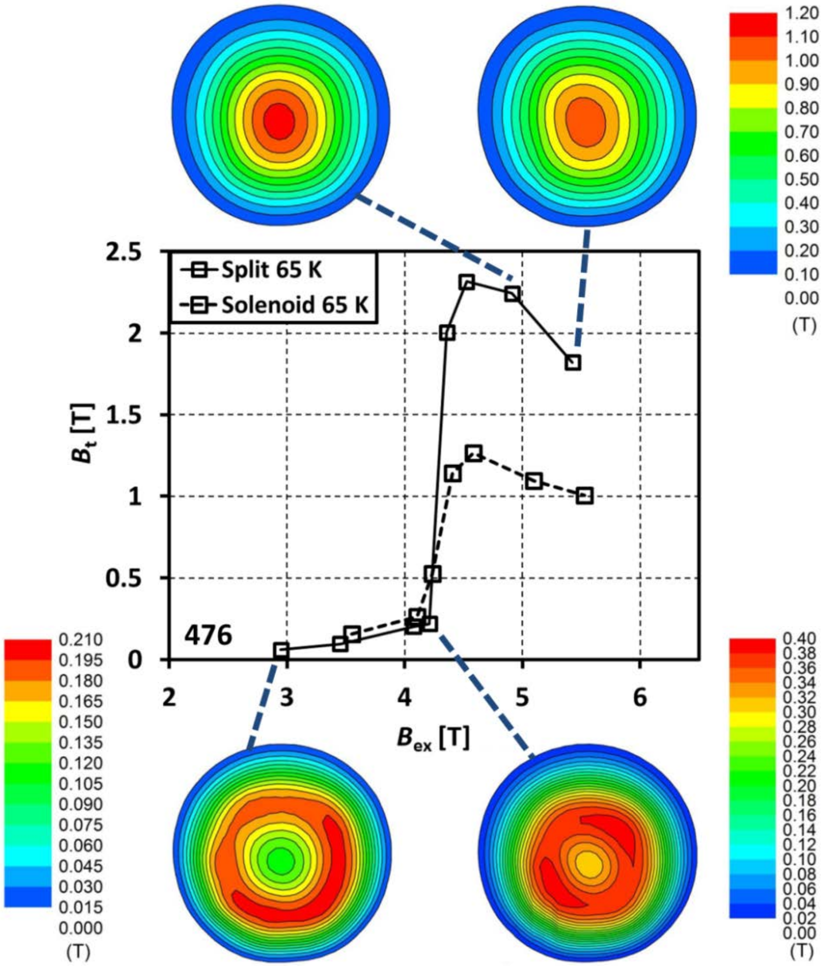
Two $GdBa_2Cu_3O_{7-\delta}$ (15wt% Ag) samples:
30 mm diameter, 15 mm thickness

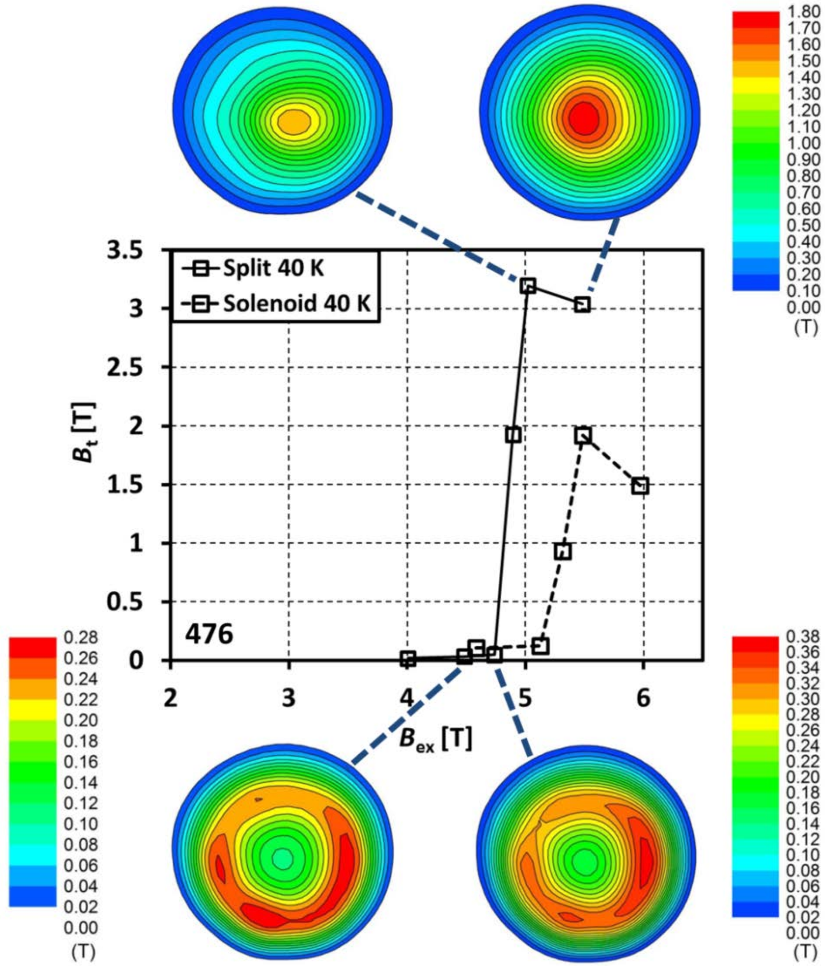
#476 sample B_t = 6 T (40 K) 3.11 T (65 K)
#477 sample B_t = 5.44 T (40 K) 2.02 T (65 K)



Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **29** (2016) 074003

Split Coil PFM with an Iron Yoke





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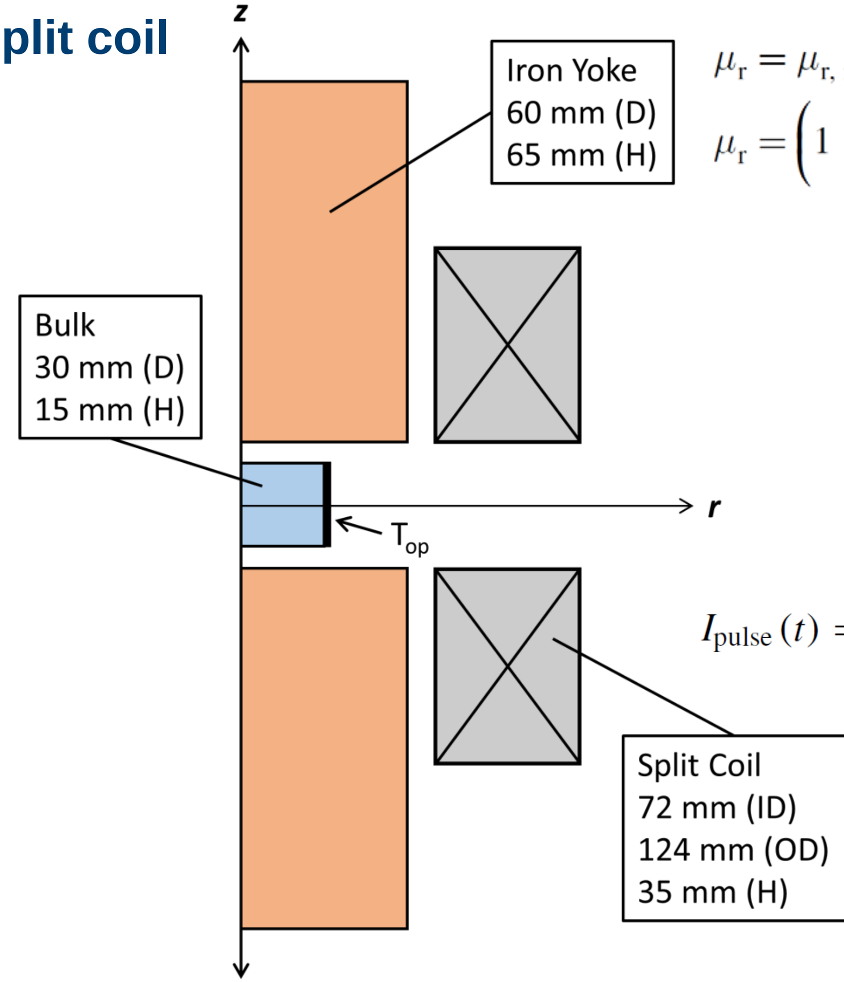
Split Coil PFM with an Iron Yoke

Split coil

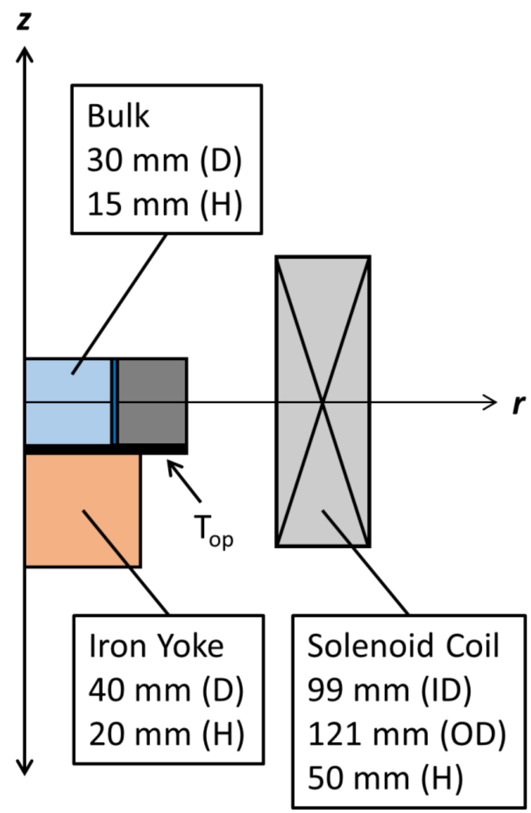


$$\mu_r = \mu_{r,\,\max} \text{ for } B < \mu_0 M_{sat}$$

$$\mu_r = \left(1 + \frac{M_{sat}}{H}\right) \text{ for } B \geqslant \mu_0 M_{sat}$$

$$I_{pulse}(t) = N \cdot I_0 \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right)$$

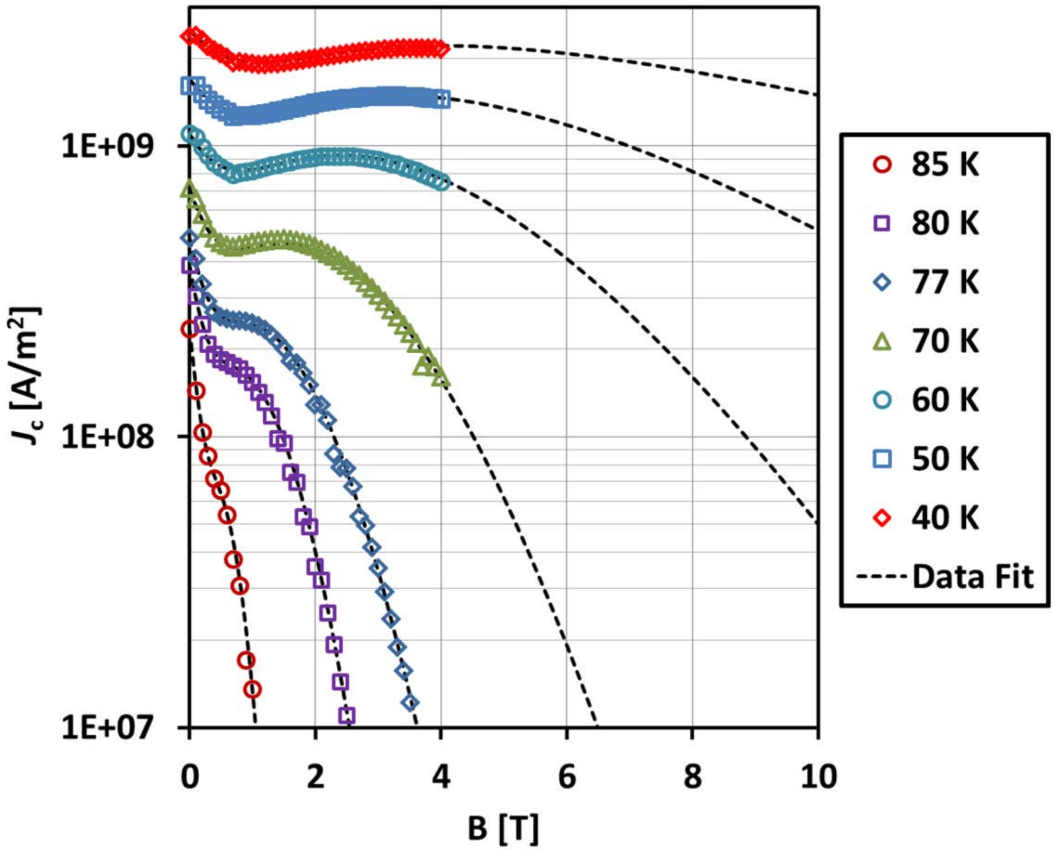
Solenoid coil

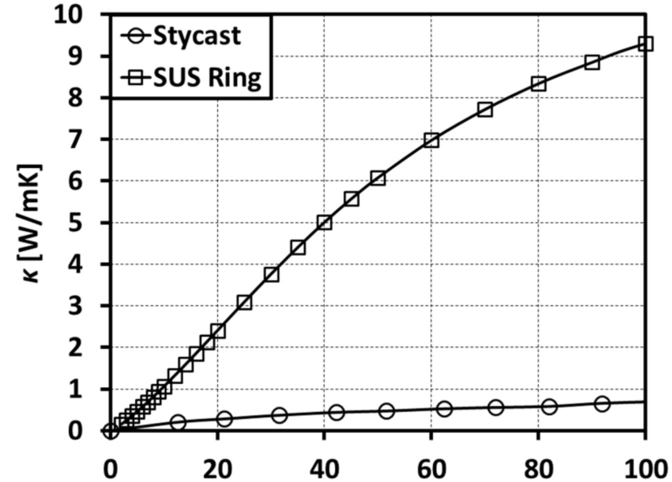


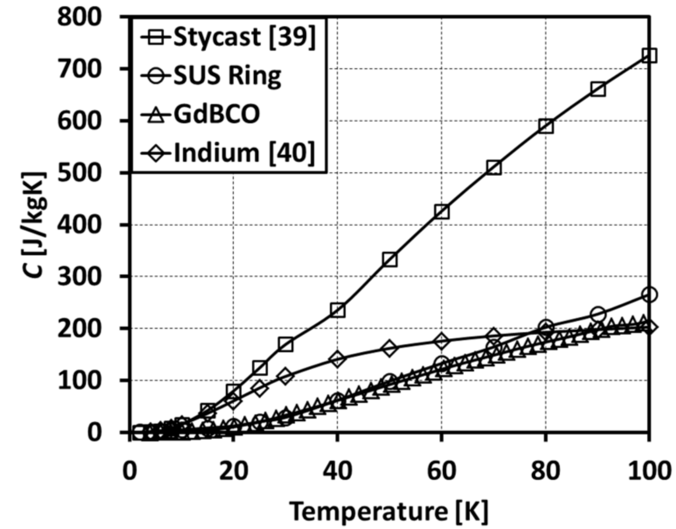
Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **29** (2016) 074003

Split Coil PFM with an Iron Yoke

$$J_c(B) = J_{c1}\exp\left(-\frac{B}{B_L}\right) + J_{c2}\frac{B}{B_{max}}\exp\left[\frac{1}{y}\left(1-\left(\frac{B}{B_{max}}\right)^{y}\right)\right]$$

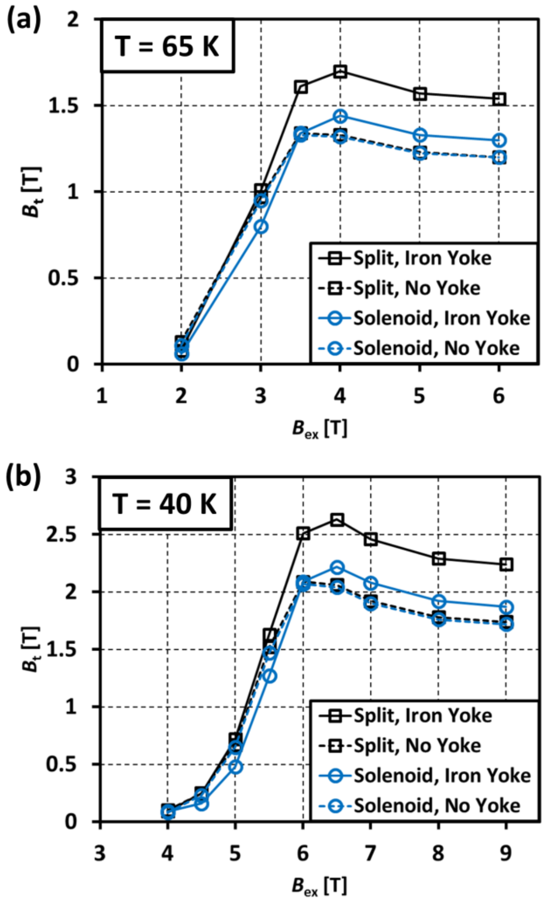




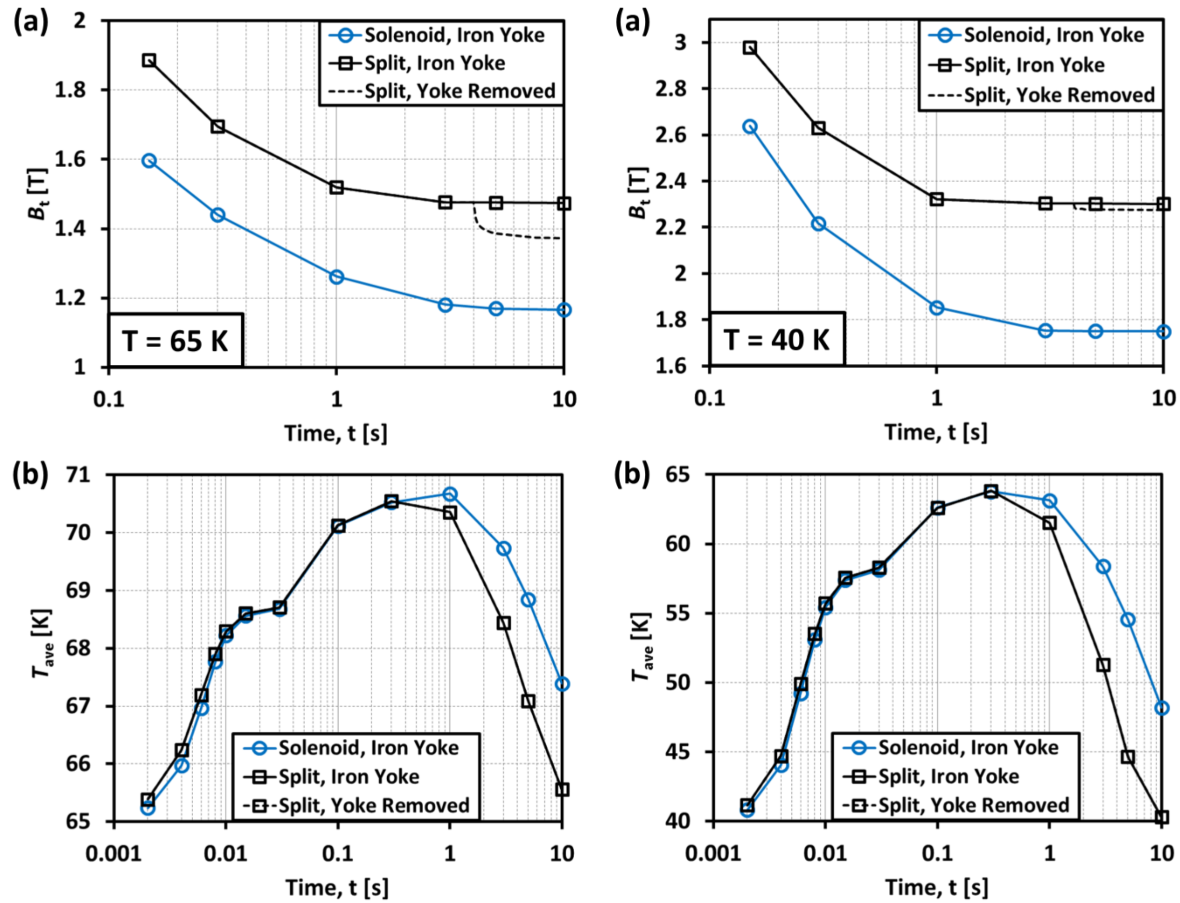


Split Coil PFM with an Iron Yoke

Trapped field @ t = 300 ms



Trapped field & T_{ave} with time, incl. yoke removed



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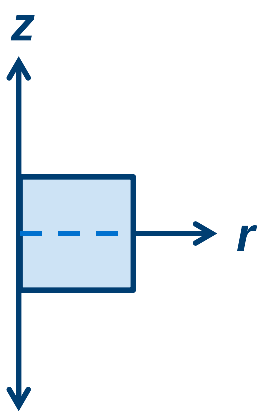
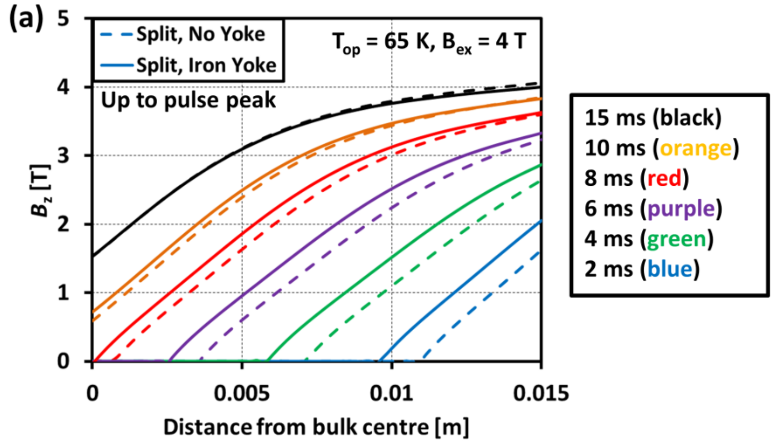
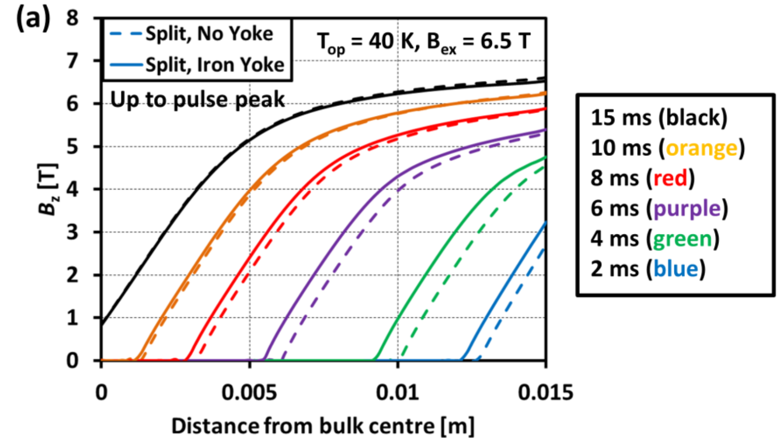
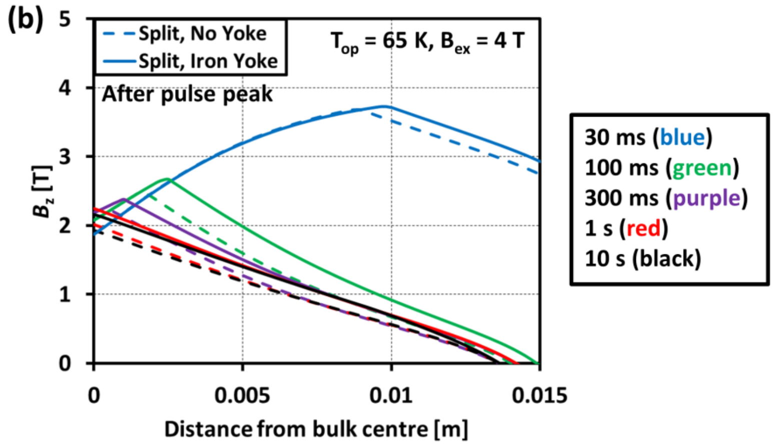
Split Coil PFM with an Iron Yoke

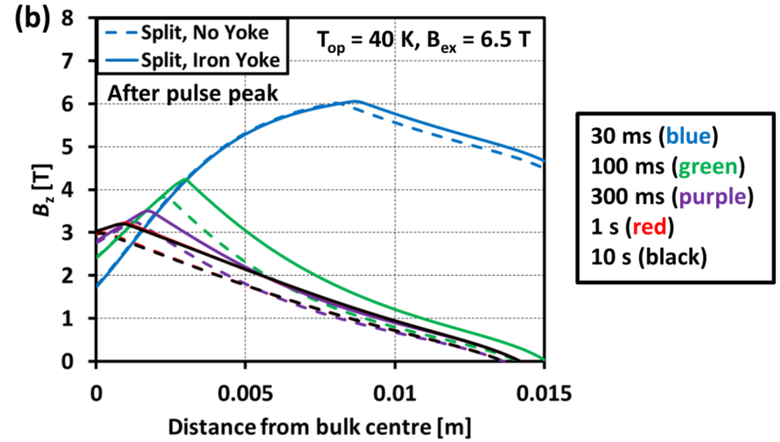
Magnetic flux entry & exit during & after pulse – split coil with & without iron yoke











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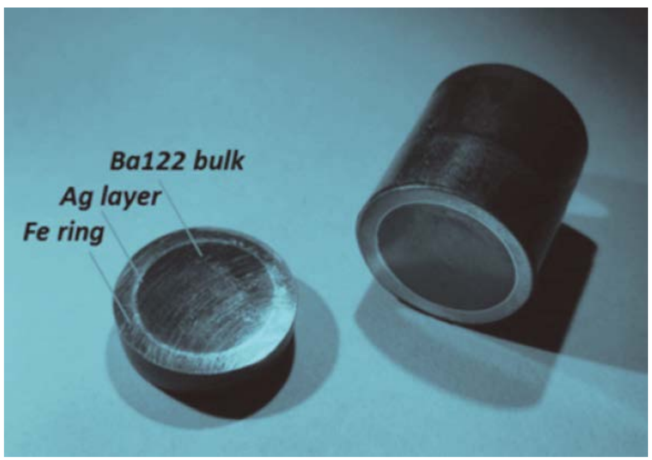
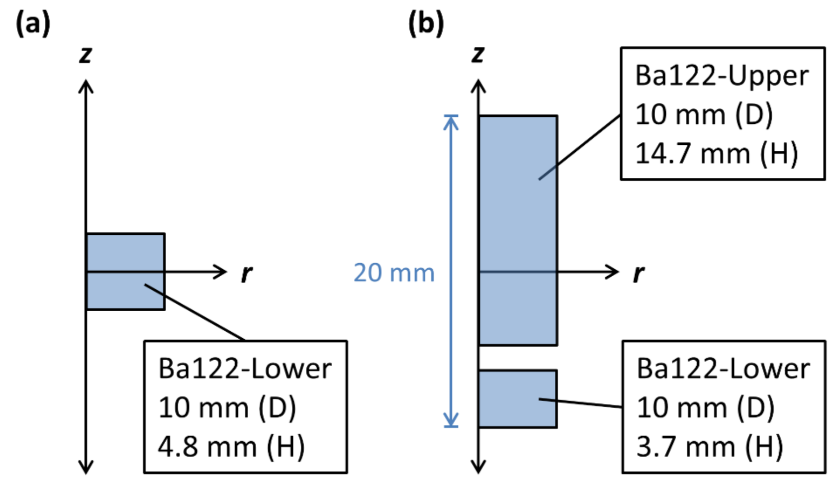
Case study #3:

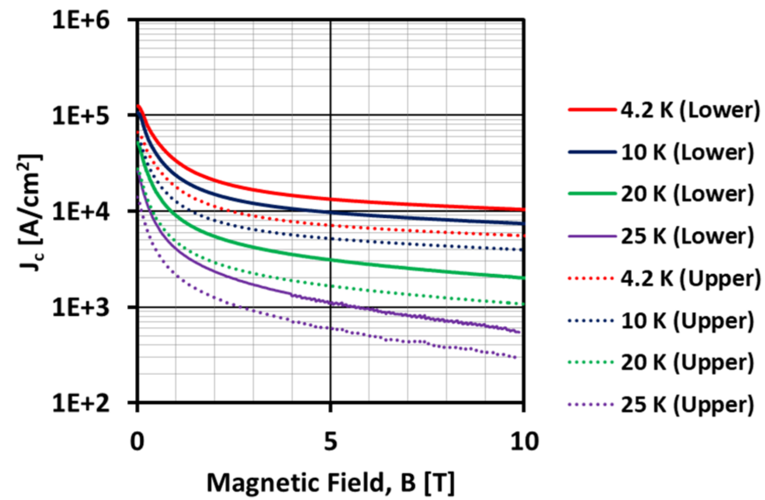
Field cooling (FC) magnetisation of iron-pnictide (Ba122) bulks

Modelling Iron-Pnictide Bulks

- Weiss et al. demonstrated > 1 T trapped in a stack of iron-pnictide (Ba122) bulks
- Advantages:
  - Low anisotropy
  - High upper critical magnetic field (H_{c2})
  - High homogeneity
  - Scalable, low-cost fabrication







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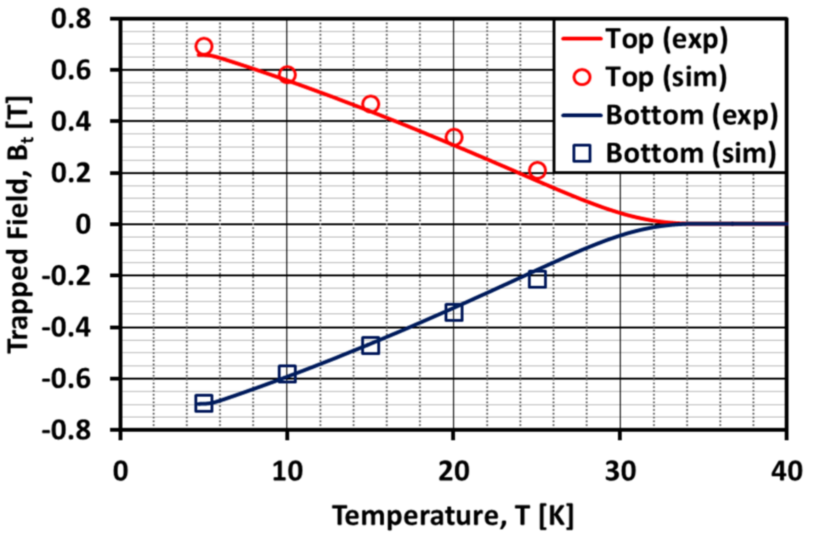
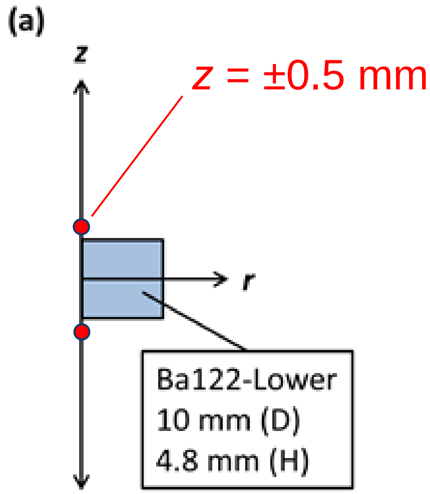
Modelling Iron-Pnictide Bulks

- Same ZFC model as before
  → approximates FC, applied field >> full penetration field

- *n* = 50
  → observed flux creep much weaker than HTS bulks





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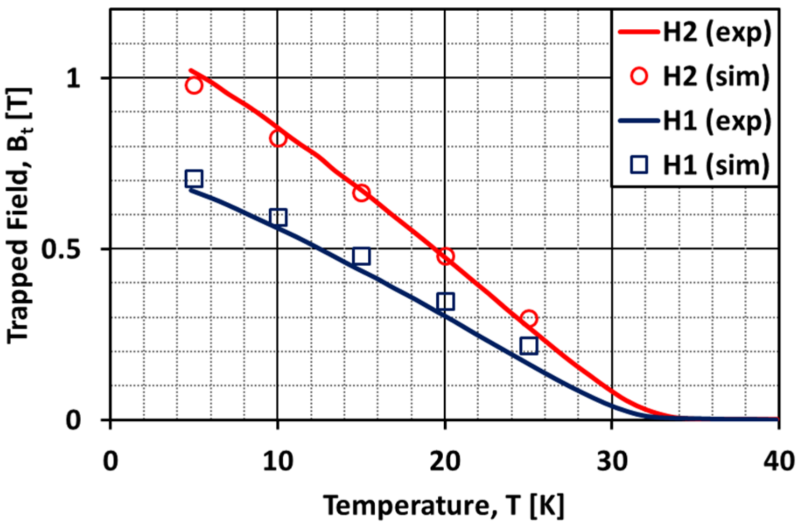

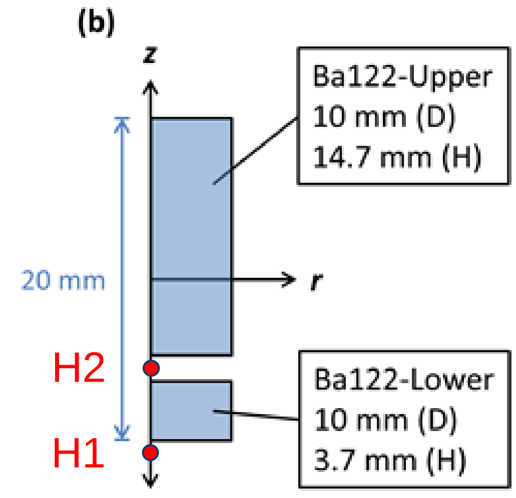

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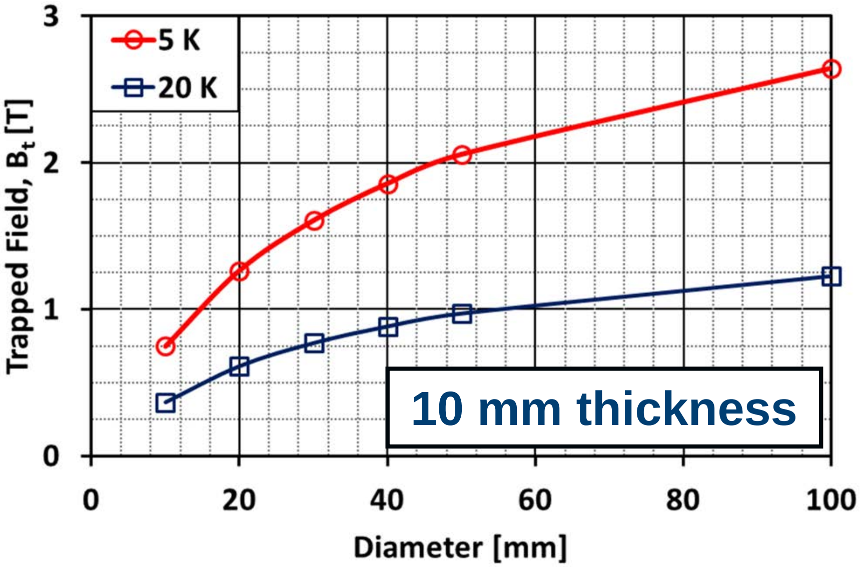
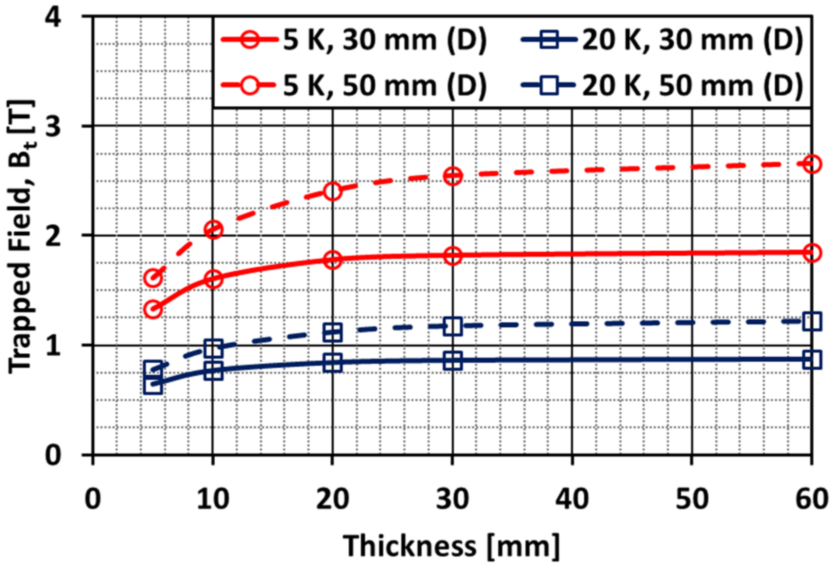
Modelling Iron-Pnictide Bulks

Influence of geometric parameters can be predicted:

Diameter dependence



Thickness dependence



- With current state-of-the-art properties, > 2 T @ 5 K (> 1 T @ 20 K) for D = 50 mm
- Appropriate aspect ratio 1-1.5 (radius : thickness)

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Summary

- **Overview of numerical modelling of magnetisation**
- **Case studies:**
  - Multi-pulse, pulsed field magnetisation (PFM) of bulk high-temperature superconductors
  - Split coil PFM with an iron yoke
  - Field cooling (FC) magnetisation of iron-pnictide (Ba122) bulks

Thank you for listening