時間依存 Ginzburg-Landau 理論

The Time-Dependent Ginzburg-Landau Theory

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1. Time-dependent Ginzburg-Landau (TDGL) Equations
2. Energy Balance
3. Vortex Flow
4. Macroscopic Modelling Simulation

Time-Dependent Ginzburg-Landau Eqs.

[A. Schmid, Phys. Kondens. Materie **5**, 302 (1966).]

- Time-dependent Ginzburg-Landau (TDGL) equation:
  - order parameter $\psi = |\psi| \exp(i\varphi)$; superfluid density $|\psi|^2$, phase φ
  - ψ normalized ($|\psi| = 1$ for zero field)

$$\tau_0\left(\frac{\partial}{\partial t} + i\frac{2\pi}{\phi_0}\Phi\right)\psi = \xi_0^2\left(\nabla - i\frac{2\pi}{\phi_0}\boldsymbol{A}\right)^2\psi + \left(1 - |\psi|^2\right)\psi$$

	Nb	NbN
T_c	9K	16K
λ_0	85nm	200nm
ξ_0	40nm	5nm

- Current density, $\mathrm{rot\,rot}\boldsymbol{A}/\mu_0 = \boldsymbol{j}_s + \boldsymbol{j}_n$

$$\begin{cases} \boldsymbol{j}_s = \dfrac{|\psi|^2}{\mu_0\lambda_0^2}\left(\dfrac{\phi_0}{2\pi}\nabla\varphi - \boldsymbol{A}\right), \\ \boldsymbol{j}_n = \sigma_n\boldsymbol{E} = -\sigma_n\left(\dfrac{\partial\boldsymbol{A}}{\partial t} + \nabla\Phi\right) \end{cases}$$

- τ_0 : GL relaxation time
- ξ_0 : coherence length
- λ_0 : penetration depth
- Φ : scalar potential
- $\boldsymbol{A}$: vector potential
- ϕ_0 : flux quantum

Derivation and Limitation of the TDGL Equations

[N. B. Kopnin, "Theory of Nonequilibrium Superconductivity," Clarendon Press (Oxford 2001).]

- Phenomenological derivation of the TDGL equation for slow relaxation of the order parameter ψ to the equilibrium state:
  - Ginzburg-Landau free energy, F_{GL}

$$\gamma \frac{\partial \psi}{\partial t} = -\frac{\delta F_{\mathrm{GL}}}{\delta \psi^*} \xrightarrow{\text{gauge invariance}} \gamma \left(\frac{\partial}{\partial t} + i \frac{2\pi}{\phi_0} \Phi \right) \psi = -\frac{\delta F_{\mathrm{GL}}}{\delta \psi^*}$$

- Microscopic consideration leads to the limitation of the TDGL equations:
  - close to T_{c}
  - deviations from the equilibrium are small
  - the quasiparticle excitations are in equilibrium with the heat bath
  - gapless superconductivity

  ➔ generalized TDGL eqn. [Kramer and Watts-Tobin, PRL 40, 1041 (1978).]

$$\tau_0 \left(1 + \Gamma^2 |\psi|^2\right)^{-1/2} \left(\frac{\partial}{\partial t} + i \frac{2\pi}{\phi_0} \Phi + \frac{\Gamma^2}{2} \frac{\partial |\psi|^2}{\partial t} \right) \psi = \xi_0^2 \left(\nabla - i \frac{2\pi}{\phi_0} \boldsymbol{A} \right)^2 \psi + \left(1 - |\psi|^2\right) \psi$$

  - further extensions to:
    d-wave, muliband (mulitigap), Hall effect,...
    ψ_1 and ψ_2 — $\gamma_1 + i\gamma_2$

Gauge for Vector *A* and Scalar *Φ* Potentials

- Time-dependent Ginzburg-Landau (TDGL) equation:

$$\tau_0 \left(\frac{\partial}{\partial t} + i \frac{2\pi}{\phi_0} \Phi \right) \psi = \xi_0^2 \left(\nabla - i \frac{2\pi}{\phi_0} \boldsymbol{A} \right)^2 \psi + \left(1 - |\psi|^2\right) \psi$$

- Gauge invariance

$$\begin{cases} \psi \to \psi e^{i\chi}, \\ \boldsymbol{A} \to \boldsymbol{A} + (\phi_0 / 2\pi) \nabla \chi, \\ \Phi \to \Phi - (\phi_0 / 2\pi) \partial \chi / \partial t \end{cases}$$

- Gauge choices
  - $\Phi = 0$
  - $\mathrm{div}\, \boldsymbol{A} = 0$
  - $\Phi = -(1/\mu_0 \sigma_{\mathrm{n}})\, \mathrm{div}\, \boldsymbol{A}$
  - $\Phi = -(\omega/\mu_0 \sigma_{\mathrm{n}})\, \mathrm{div}\, \boldsymbol{A}$ with $\omega \geq 0$
  - thin wire approximation, $\boldsymbol{A} = \boldsymbol{r} \times \boldsymbol{B}_{\mathrm{a}} / 2$

Energy Balance of the Electromagnetic Energy

- Electromagnetic energy, F_{em}

$$F_{\text{em}} = \frac{1}{2\mu_0}\boldsymbol{B}^2 + \frac{\varepsilon_0}{2}\boldsymbol{E}^2$$

$$\boldsymbol{H} = \boldsymbol{B}/\mu_0, \quad \boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla\times\boldsymbol{H} = \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}$$

  - time derivative: Poynting's theorem

$$\frac{\partial F_{\text{em}}}{\partial t} = \boldsymbol{H}\cdot\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{E}\cdot\frac{\partial \boldsymbol{D}}{\partial t} = -\nabla\cdot\left(\boldsymbol{E}\times\boldsymbol{H}\right) - \boldsymbol{E}\cdot\boldsymbol{j}$$

Poynting's vector

Energy Balance of the EM and GL Free Energy

- Free energy, $F_{\text{em}} + F_{\text{GL}}$ [A. Schmid, Phys. Kondens. Materie **5**, 302 (1966).]

$$F_{\text{em}} = \frac{1}{2\mu_0}\boldsymbol{B}^2 + \frac{\varepsilon_0}{2}\boldsymbol{E}^2,$$ electromagnetic energy

$$F_{\text{GL}} = \mu_0 H_c^2\left[\xi_0^2\left|\left(\nabla - i\frac{2\pi}{\varphi_0}\boldsymbol{A}\right)\psi\right|^2 - |\psi|^2 + \frac{1}{2}|\psi|^4\right]$$ Ginzburg-Landau free energy

  - time derivative:

$$\frac{\partial}{\partial t}\left(F_{\text{em}} + F_{\text{GL}}\right) = -\nabla\cdot\boldsymbol{S}_E - W,$$

  - energy current

$$\boldsymbol{S}_E = \boldsymbol{E}\times\boldsymbol{H} + 2\mu_0 H_c^2 \xi_0^2 \,\mathrm{Re}\left[\left(\nabla\psi^* + i\frac{2\pi}{\phi_0}\boldsymbol{A}\psi^*\right)\left(\frac{\partial\psi}{\partial t} + i\frac{2\pi}{\phi_0}\Phi\psi\right)\right],$$

Poynting's vector

GL energy current

  - dissipation

$$W = \sigma_n \boldsymbol{E}^2 + 2\mu_0 H_c^2 \tau_0 \left|\left(\frac{\partial}{\partial t} + i\frac{2\pi}{\phi_0}\Phi\right)\psi\right|^2$$

Ohmic dissipation

dissipation due to the order parameter relaxation

2D Vortex Flow: Straight Vortices

[L. P. Gor'kov and N. B. Kopnin, Sov. Phys. Usp. **18**, 496 (1976).]

- Two dimensional vortex flow without pinning for straight vortices
  - GL solution for steady state: $\psi_0(\boldsymbol{r})$
  - TDGL solution for uniform vortex flow: $\psi(\boldsymbol{r},t) = \psi_0(\boldsymbol{r}-\boldsymbol{v}_0 t)$
    expansion in powers of the mean flow speed $\boldsymbol{v}_0$

$\partial/\partial t \to \boldsymbol{v}_0 \cdot \nabla$

➔ equation of motion for vortices (derived from TDGL!):

$$\eta \boldsymbol{v}_0 = \phi_0 \boldsymbol{j} \times \hat{z}$$

viscous drag force — Lorentz force

- vortex-flow conductivity, $\sigma_f = \eta/\phi_0 B$ ($\boldsymbol{j} = \sigma_f \boldsymbol{E}$)
  vortex-flow dissipation, $W = \sigma_f \langle \boldsymbol{E} \rangle^2$

$$W = \sigma_n \boldsymbol{E}^2 + 2\mu_0 H_c^2 \tau_0 \left| \left(\frac{\partial}{\partial t} + i \frac{2\pi}{\phi_0} \Phi \right) \psi \right|^2$$

$\sigma_f = \sigma_n B_{c2}/B$
[J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).]

substantial contribution from the order-parameter relaxation term
[M. Tinkham, PRL **13**, 804(1964).]

➔ vortex-flow Hall effect ($\gamma \to \gamma_1 + i\gamma_2$):

$$\eta \boldsymbol{v}_0 + \alpha \boldsymbol{v}_0 \times \hat{z} = \phi_0 \boldsymbol{j} \times \hat{z}$$

[A. T. Dorsey, PRB **46**, 8376 (1992).]

3D Vortex Flow: Bent Vortices

[L. P. Gor'kov and N. B. Kopnin, Sov. Phys. Usp. **18**, 496 (1976).]

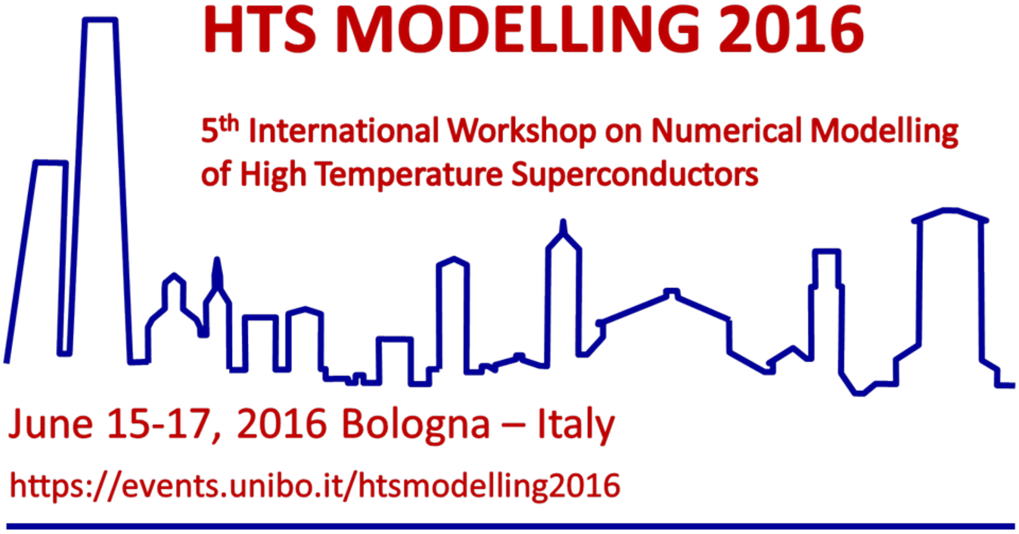
- Three dimensional vortex flow without pinning for bent vortices
  - GL solution for steady state: $\psi_0(\boldsymbol{r})$
  - TDGL solution for vortex flow: $\psi(\boldsymbol{r}, z, t) = \psi_0(\boldsymbol{r}-\boldsymbol{r}_0(z,t))$
    expansion in powers of $\partial \boldsymbol{r}_0/\partial t$ and $\partial \boldsymbol{r}_0/\partial z$

➔ equation of motion for vortices (derived from TDGL!):

$$\eta \frac{\partial \boldsymbol{r}_0}{\partial t} = \varepsilon_1 \frac{\partial^2 \boldsymbol{r}_0}{\partial z^2} + \phi_0 \boldsymbol{j} \times \hat{z}$$

viscous drag force — vortex line tension — Lorentz force

HTS Modelling Workshop



A very complex behaviour is obtained when dealing with HTS materials due to high non-linearity and hysteresis, strong anisotropy, temperature dependence, high aspect ratio and complex composite structure of practical wires and tapes. Such a complex behaviour raises new challenges in the development of reliable modelling tools and requires a specialized research effort to be effectively dealt with.

Lausanne (2010), Cambridge (2011), Barcelona (2012), and Bratislava (2014)

Summary

Time-dependent Ginzburg-Landau (TDGL) theory:

- Although TDGL equations are strictly valid only in a state close to the equilibrium, they give reasonable pictures for the wide variety of nonequilibrium phenomena (e.g., vortex flow).
- Poynting's theorem for the electromagnetic energy is naturally extended to the energy balance including the Ginzburg-Landau free energy (TDGL and Maxwell equations).
- Dissipation due to the order parameter relaxation contributes to the total dissipation, in addition to the Ohmic dissipation.
- Equations of motion for vortices (without pinning centers) are naturally derived from the TDGL equations.

References:

- N. B. Kopnin, "Theory of Nonequilibrium Superconductivity," Clarendon Press (Oxford 2001).
- A. Schmid, "A time dependent Ginzburg-Landau equations and its application to the problem of resistivity in the mixed state," Phys. Kond. Materie 5, 302 (1966).
- L. P. Gor'kov and N. B. Kopnin, "Vortex motion and resistivity of type-II superconductors in a magnetic field," Sov. Phys. Usp. **18**, 496 (1976).
- A. T. Dorsey, "Vortex motion and the Hall effect in type-II superconductors: A time-dependent Ginzburg-Landau theory approach," PRB **46**, 8376 (1992).