TDGL Simulation on the Motion of Flux Lines in a Thin Superconducting Wire in a Transverse Magnetic Field

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The 65th JSAP Spring Meeting

March 17, WASEDA University, Nishi-Waseda Campus

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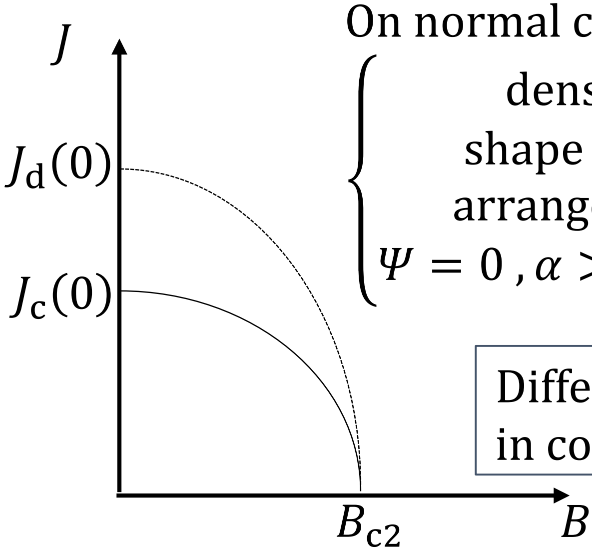
1. Background

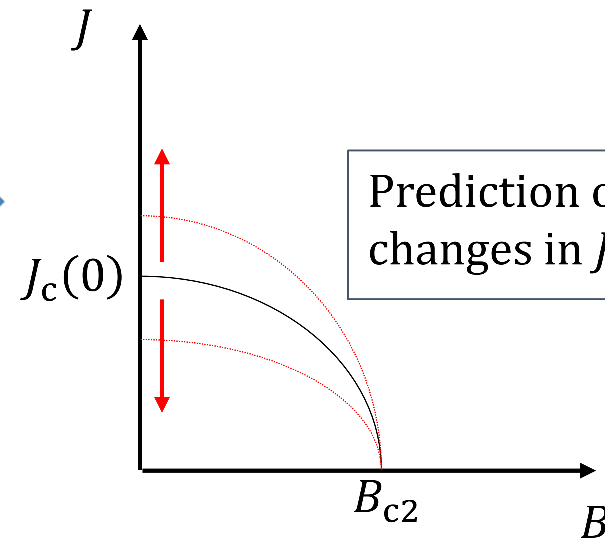
J : current density
J_d : destruction current density
J_c : critical current density
B : magnetic field
B_{c2} : critical magnetic field
Ψ : order parameter



On normal conductor(pin)…

- density
- shape / size
- arrangement
- $\Psi = 0 \, , \alpha > 0 \;\; \beta > 0$

Differences in conditions



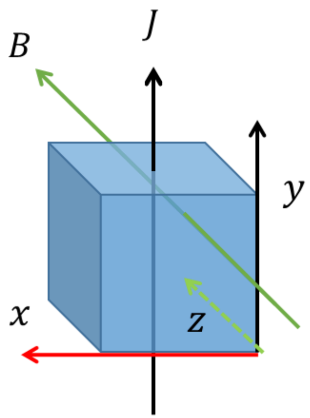
Prediction of changes in J_c

Which condition of the pin affects J_c?

Improvement of computer processing performance in recent years

It has become easier to investigate the relationship by solving TDGL simulation

2. In present study



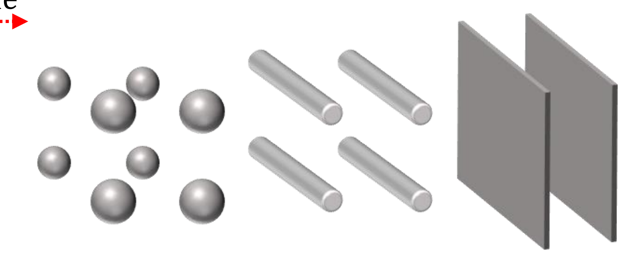
Transverse magnetic field

On normal conductor

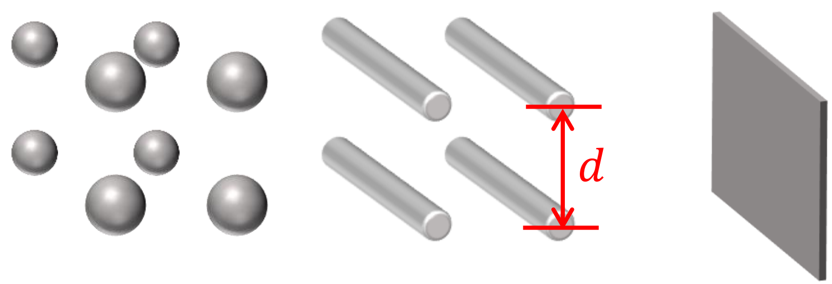
- density
- shape / size
- arrangement
- boundary condition

For example →

sphere cylinder plane



sphere | cylinder | plane



Investigate E-J and J_c-B properties.

TDGL equation

- Ginzburg-Landau(GL) equation
  - Phenomenological theory to explain superconductivity
  - Calculate order parameter Ψ
  - $|\Psi|^2$:Superconducting electron density

$$\begin{cases} \frac{1}{2m^*}(-\mathrm{i}\hbar\boldsymbol{\nabla} + 2eA)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0 \\ \boldsymbol{j} = \frac{\mathrm{i}\hbar e}{m^*}(\Psi^*\boldsymbol{\nabla}\Psi - \Psi\boldsymbol{\nabla}\Psi^*) - \frac{4e^2}{m^*}|\Psi|^2\boldsymbol{A} \end{cases}$$

- Time Dependent GL(TDGL) equation
  - GL equations with time dependency

$$\begin{cases} \gamma\left(\frac{\partial\Psi}{\partial t} + 2\mathrm{i}eV\Psi\right) + \frac{1}{2m^*}(-i\hbar\boldsymbol{\nabla} + 2e\boldsymbol{A})^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0 \\ \nu\left(\frac{\partial\boldsymbol{A}}{\partial t} + \boldsymbol{\nabla}V\right) + \frac{1}{\mu_0}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{A} - \frac{\mathrm{i}\hbar e}{m^*}(\Psi^*\boldsymbol{\nabla}\Psi - \Psi\boldsymbol{\nabla}\Psi^*) + \frac{4e^2}{m^*}|\Psi|^{\boldsymbol{2}}\boldsymbol{A} = 0 \end{cases}$$

3. Normalize TDGL equation

- Equations

$$\begin{cases} \dfrac{\partial \Psi}{\partial t} + \mathrm{i} V \Psi + (-\mathrm{i}\nabla - \boldsymbol{A})^2 \Psi - \Psi + |\Psi|^2 \Psi = 0 & (1) \\ \sigma \nabla^2 V = \dfrac{\mathrm{i}}{2}(\Psi^* \nabla^2 \Psi - \Psi \nabla^2 \Psi^*) - \nabla \cdot (|\Psi|^2 \boldsymbol{A}) & (2) \\ \Psi = 0 & (3) \end{cases}$$

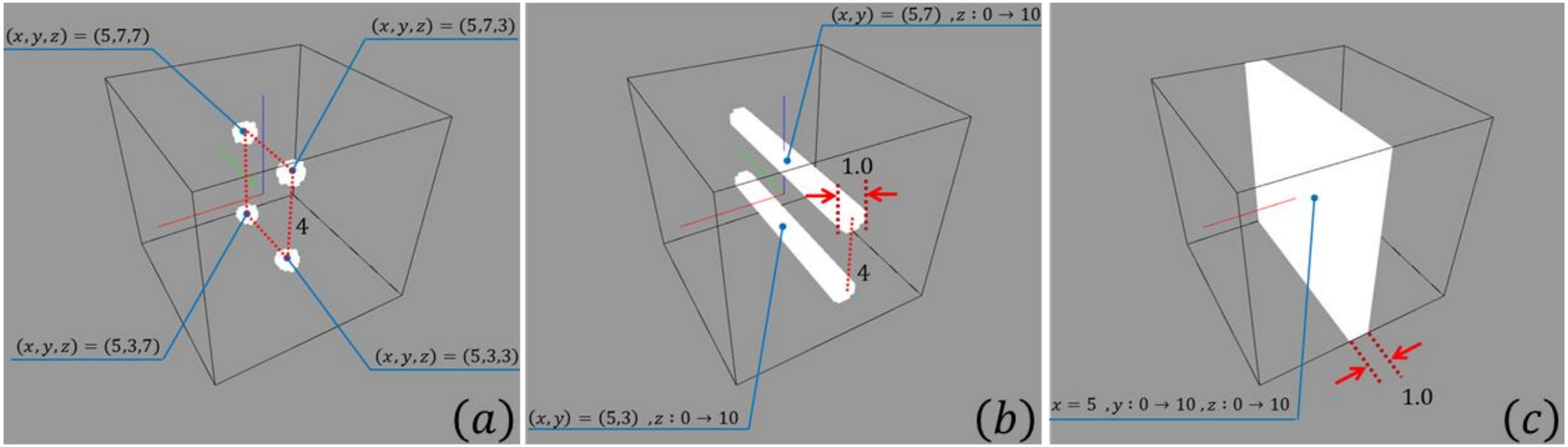
(1) Ψ in superconducting region

(2) Scalar potential V

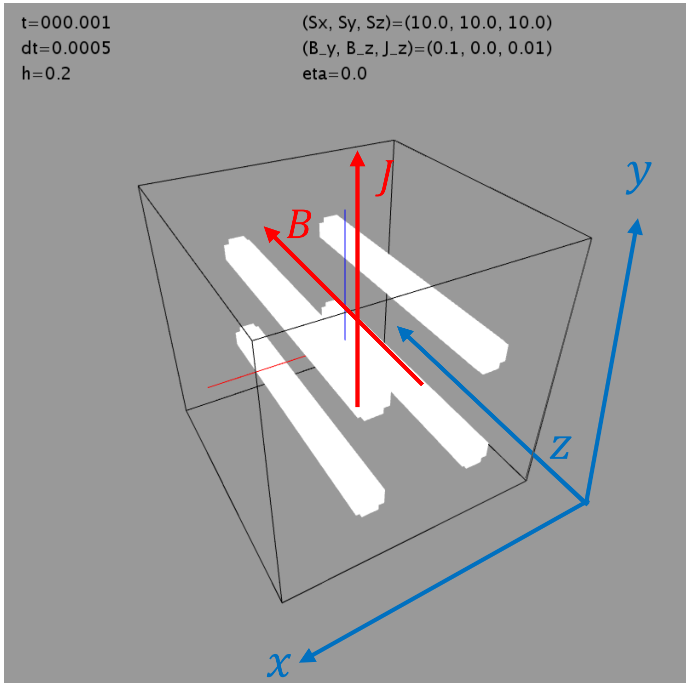
(3) Ψ in normal region

4. Simulation model

- Assuming in vacuum
- Consider a cube space(side length: 10ξ)
- Mesh size : $50 \times 50 \times 50$



Conditions (thin wire)



■ Current density J, Magnetic flux density B

- $\boldsymbol{J} = J_y \boldsymbol{i}_y, \quad \boldsymbol{B} = B_z \boldsymbol{i}_z \quad (\therefore \boldsymbol{J} \perp \boldsymbol{B})$
- $\boldsymbol{A} = B_x \boldsymbol{i}_y$

  ($\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z$: Unit vector of each axis)

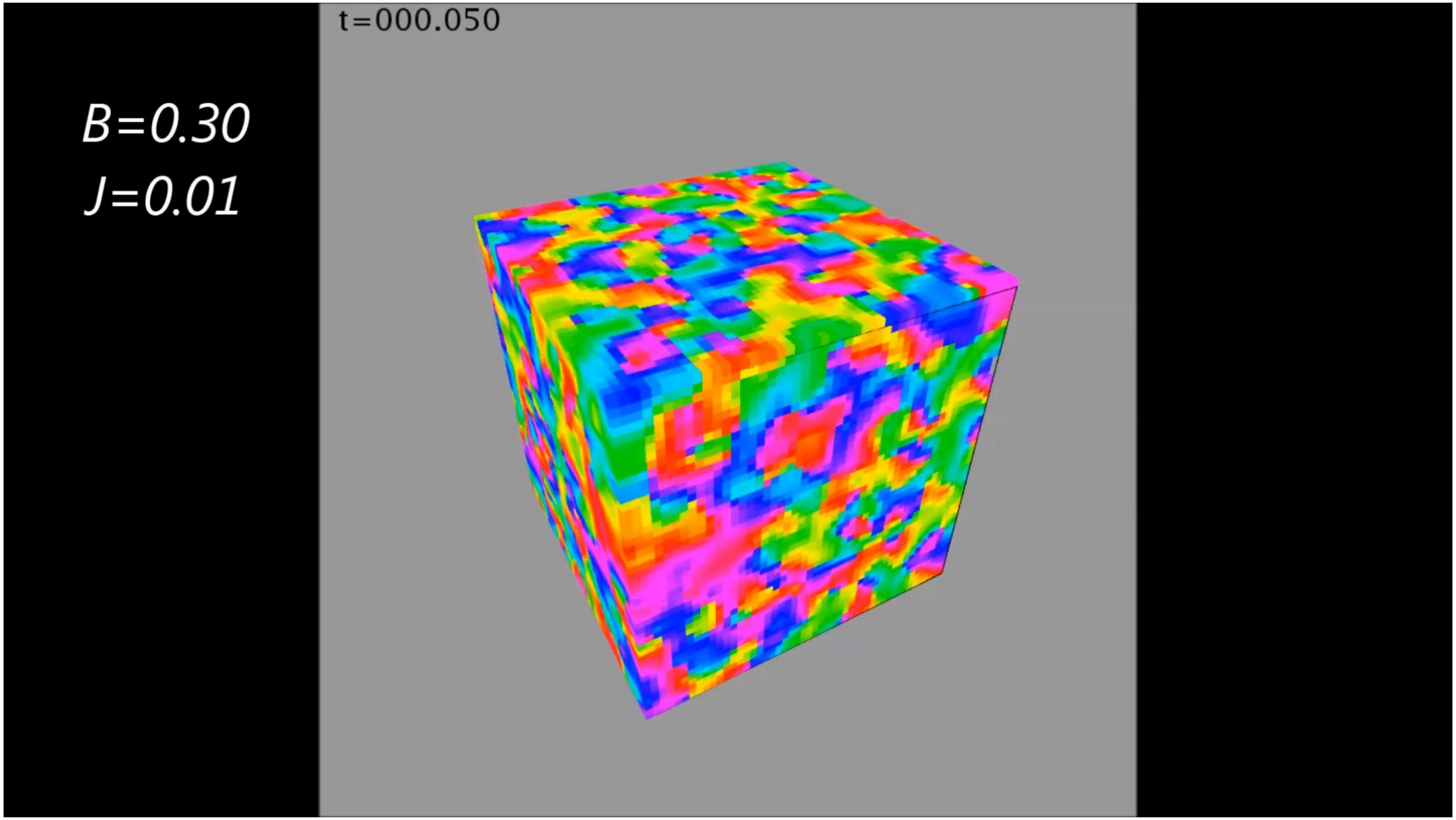
■ Initial conditions

- $\Psi(t=0) = \cos\theta + \mathrm{i}\sin\theta$
- $V(t=0) = -Jy/\sigma$

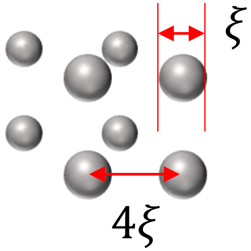
■ Boundary conditions

- $\boldsymbol{n} \cdot (\nabla\Psi + \mathrm{i}\boldsymbol{A}\Psi) = 0$
- $\nabla V = -\boldsymbol{J}/\sigma$

※$\boldsymbol{n}$: A unit vector perpendicular to the plane


t=000.050
B=0.30
J=0.01

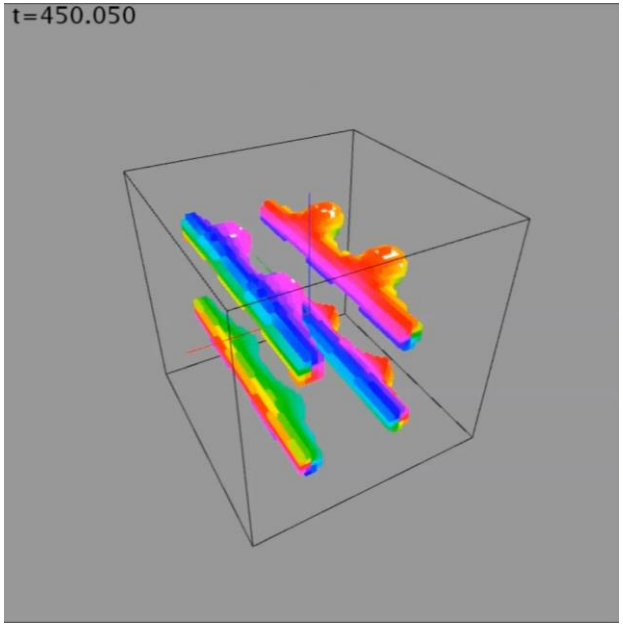
5. E-J property

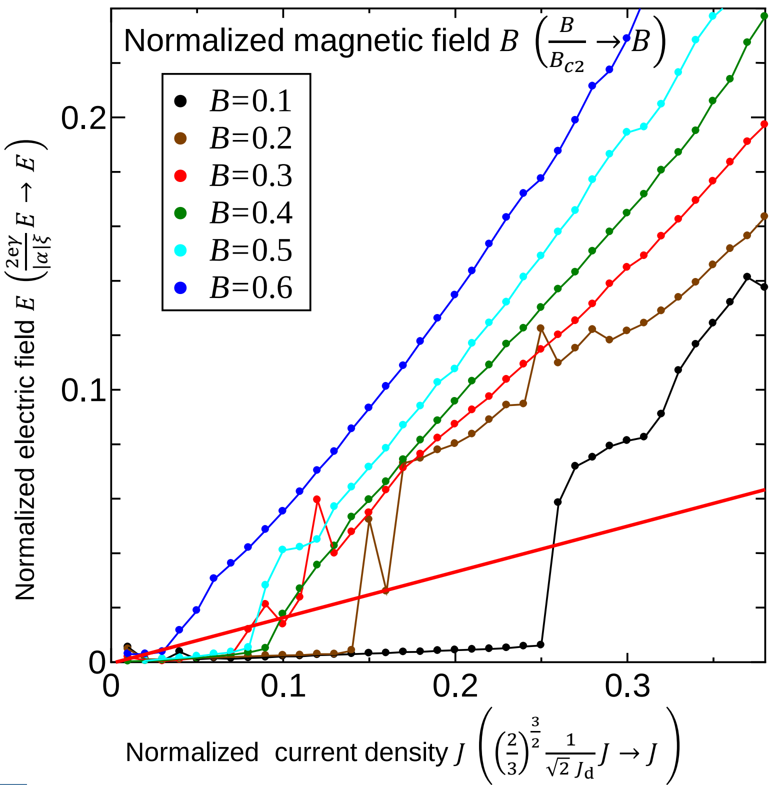


Critical current density J_c is defined using a resistance standard as indicated by a red straight line in the E-J property

$\Psi = 0$

$\boldsymbol{E} = -\boldsymbol{\nabla} V$



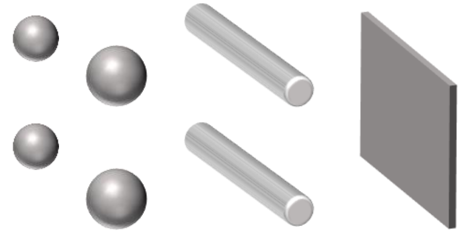


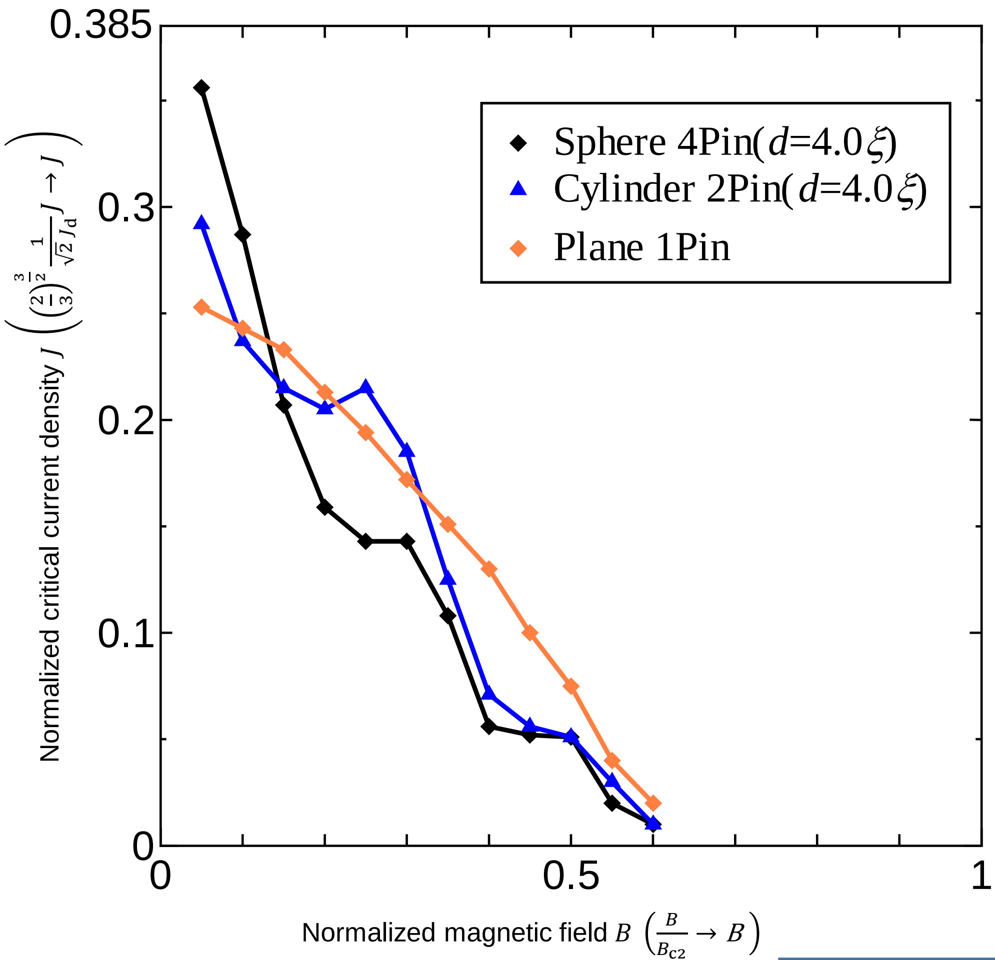
Super : $\frac{\partial \Psi}{\partial t} + iV\Psi + (-i\nabla - \boldsymbol{A})^2 \Psi - \Psi + |\Psi|^2 \Psi = 0$

Normal : $\Psi = 0$

In each E-J property, it is possible to confirm the rise of the electric field.

J_c-B property



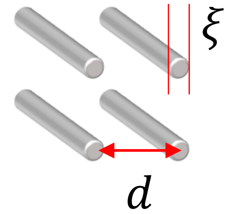


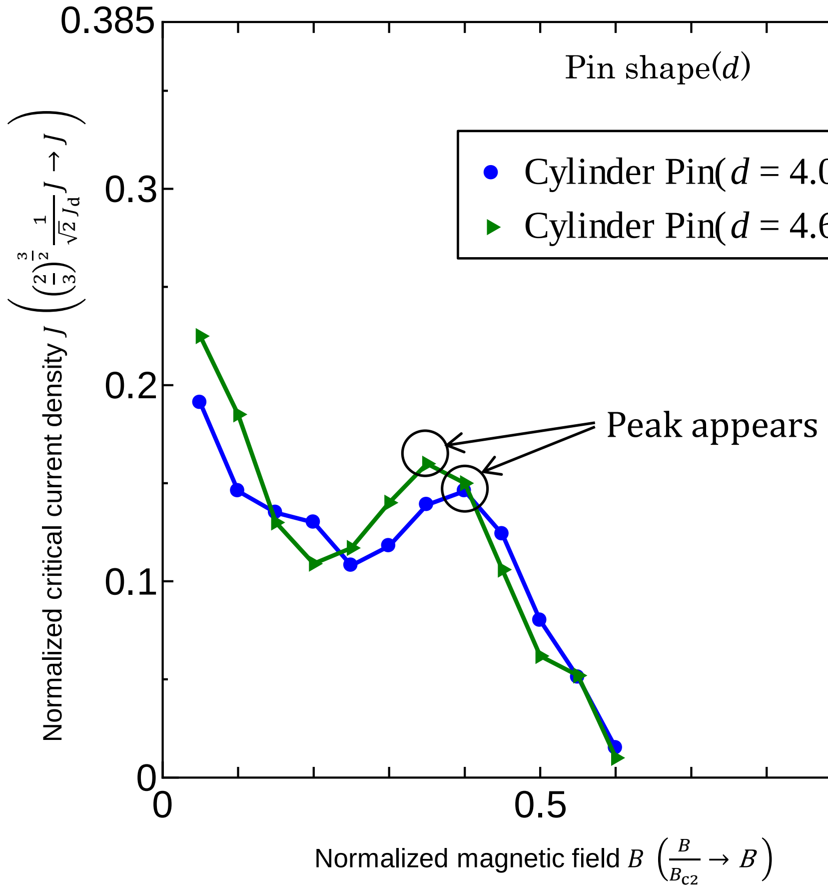
- Each J_c-B property has a different tendency
- It is confirmed that the peak appears at "Cylinder 2Pin".
- J_c monotonically decreases as B increases at "Plane 1Pin"

Super : $\frac{\partial \Psi}{\partial t} + \mathrm{i}V\Psi + (-\mathrm{i}\nabla - \boldsymbol{A})^2\Psi - \Psi + |\Psi|^2\Psi = 0$

Normal : $\Psi = 0$

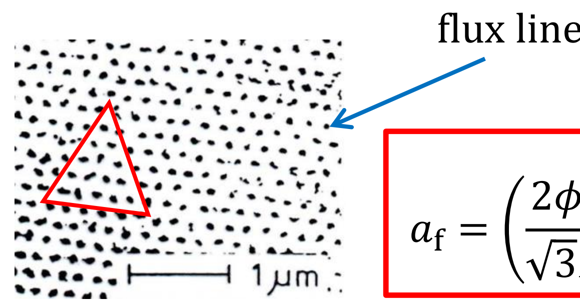
J_c-B property





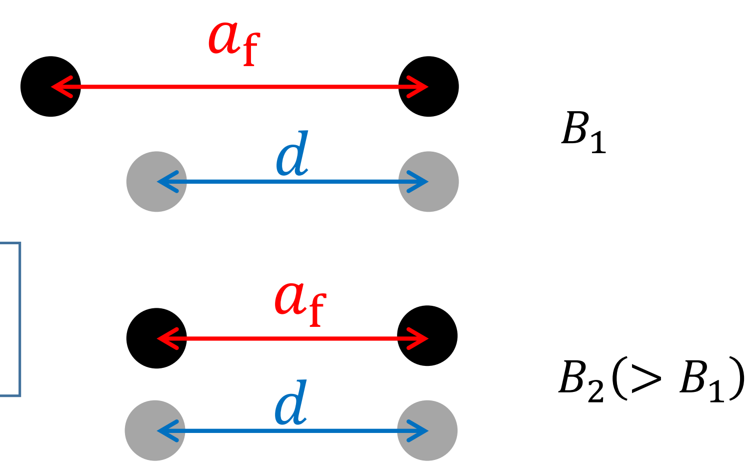
- It is confirmed that the peak appears in the J_c-B property.

※Peak effect



$$a_f = \left(\frac{2\phi_0}{\sqrt{3}B}\right)^{\frac{1}{2}} \propto \frac{1}{\sqrt{B}}$$

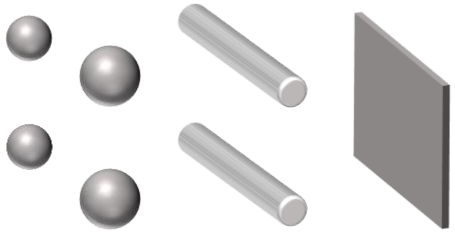
Dr. B. Obst in Researarch Center in Karlsruhe

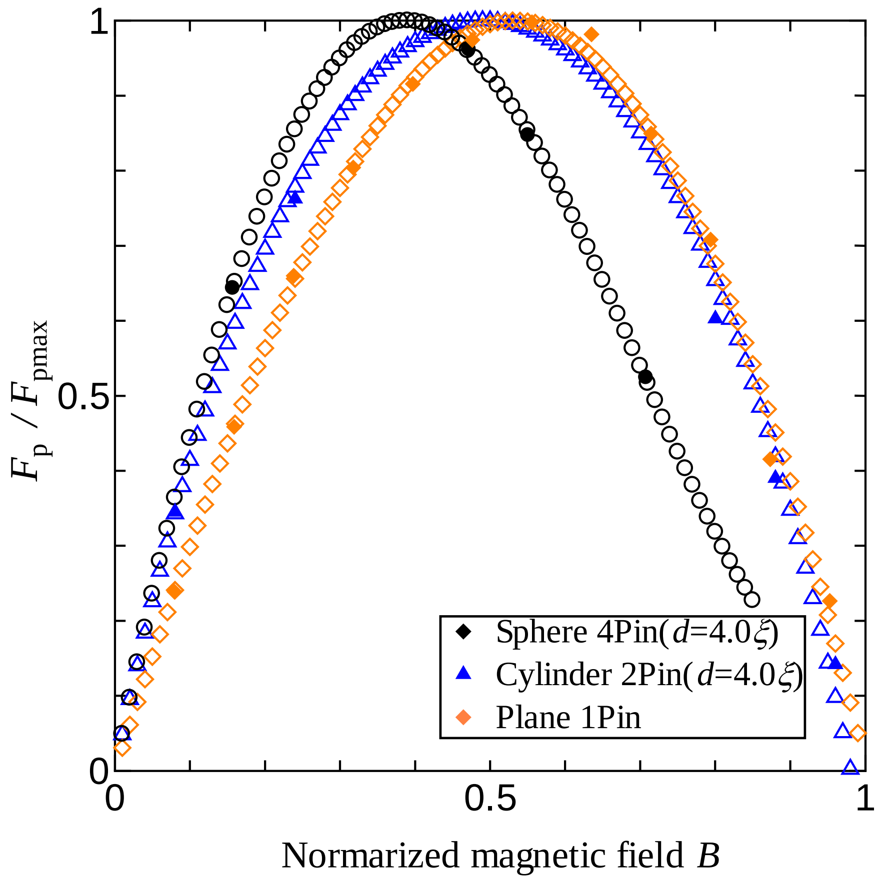


Super $: \frac{\partial \Psi}{\partial t} + iV\Psi + (-i\nabla - \boldsymbol{A})^2\Psi - \Psi + |\Psi|^2\Psi = 0$

Normal$: \Psi = 0$

Scaling law of pin force density





$$F_p \,/\, F_{pmax} = (B \,/ B_{c2})^{\gamma}(1-(B \,/ B_{c2}))^{\delta}$$

Plane

γ-> 1.12, δ-> 0.99

Cylinder

γ->0.92, δ->0.98

Sphere

γ-> 1.10, δ-> 1.75

7. Conclusions

・The J_c-B property was clarified by solving the simplified three - dimensional TDGL equation by simulation.

・Significant peak effect was confirmed in cylindrical pin.

・Plane pin shows largest pinning effect among of all.

・It may be possible in the future to discuss macroscopic features such as the scale rule of J_c by performing large scale calculations.