研　究

Optical Matrix Multiplication by Digital Binary Representation of Bipolar Numbers Using Liquid Crystal Logic Elements

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Liquid crystal logic elements have been developed for applications in optical computing. One-dimensional spatial light modulators comprising multiples of logic elements were designed, fabricated and used in binary digital representations of numbers with signs. Two spatial light modulators were crossed in the optical system and matrix multiplication performed by an outer-product operation.

1. Introduction

For handling bipolar numbers, a processor partitioning the elements into positive and negative parts or biasing to assure non-negative elements was previously suggested by Goodman et al.,[1] and described by Casasent et al.[2,3] An engagement array processor using the two's complement binary representation was developed by Bocker et al.,[4] where the negative number in matrices or vectors was converted into two's complement binary number. Also, a sign-magnitude representation processor in which only the magnitude data processed on optical system was necessary in the external electronic digital logic for a sign bit decision.[5] In this paper, we describe a processor of fully optical sign-magnitude representations for bipolar numbers using liquid crystal devices and used in matrix multiplication based on the outer product decomposition initiated by Athale et al.[6] Each matrix element is converted into a binary word comprising a sign bit (0 for a plus sign and 1 for minus) and magnitude bits of the absolute value. As an illustration, two decimal numbers (-3 and 2) can be represented as $(1\ 1\ 1)$ and $(0\ 1\ 0)$ in binary, respectively. Consider the outer product of two 1×2 vectors represented in 3-bit

binary words as follows:
$$\boldsymbol{a} = (a_1{}^s\ a_1{}^1\ a_1{}^0\quad a_2{}^s\ a_2{}^1\ a_2{}^0),$$
$$\boldsymbol{b} = (b_1{}^s\ b_1{}^1\ b_1{}^0\quad b_2{}^s\ b_2{}^1\ b_2{}^0), \qquad (1)$$
where the superscript 's' indicates the sign bit. Other superscripts represent the bit-significances of magnitude bits; the subscripts are element indices. The outer product matrix $\boldsymbol{C}$ is given by

$$\boldsymbol{C} = \boldsymbol{a}^t \times \boldsymbol{b}$$

$$= \begin{pmatrix} a_1{}^s \\ a_1{}^1 \\ a_1{}^0 \\ a_2{}^s \\ a_2{}^1 \\ a_2{}^0 \end{pmatrix} (b_1{}^s\ b_1{}^1\ b_1{}^0\quad b_2{}^s\ b_2{}^1\ b_2{}^0)$$

$$= \left(\begin{array}{ccc|ccc} a_1{}^s b_1{}^s & & & a_1{}^s b_2{}^s & & \\ & a_1{}^1 b_1{}^1 & a_1{}^1 b_1{}^0 & & a_1{}^1 b_2{}^1 & a_1{}^1 b_2{}^0 \\ & a_1{}^0 b_1{}^1 & a_1{}^0 b_1{}^0 & & a_1{}^0 b_2{}^1 & a_1{}^0 b_2{}^0 \\ \hline a_2{}^s b_1{}^s & & & a_2{}^s b_2{}^s & & \\ & a_2{}^1 b_1{}^1 & a_2{}^1 b_1{}^0 & & a_2{}^1 b_2{}^1 & a_2{}^1 b_2{}^0 \\ & a_2{}^0 b_1{}^1 & a_2{}^0 b_1{}^0 & & a_2{}^0 b_2{}^1 & a_2{}^0 b_2{}^0 \end{array}\right).$$

$$(2)$$

The transpose of vector $\boldsymbol{a}$ is represented as $\boldsymbol{a}^t$. Since there is no meaning for the multiplication between a sign bit and a magnitude bit, elements such as $a_1{}^s b_1{}^1$ and $a_1{}^1 b_1{}^s$ are dropped from the matrix of Eq. (2). The binary values at the left-top corners of the four submatrices, such as $a_1{}^s b_1{}^s$, $a_1{}^s b_2{}^s$, $a_2{}^s b_1{}^s$ and $a_2{}^s b_2{}^s$, are generated by multiplication between the sign bits, resulting in

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the signs of the elements of C. The sign-bit multiplication operations must follow exclusive OR logic operations: $1 \oplus 1 = 0$, $1 \oplus 0 = 0 \oplus 1 = 1$ and $0 \oplus 0 = 0$. The others are generated by multiplication operations between magnitude bits: the AND logic operations of $1 \times 1 = 1$ and $1 \times 0 = 0 \times 1 = 0 \times 0 = 0$.

2. Liquid Crystal Spatial Light Modulator

To optically achieve these two different logic operations in parallel for a calculation of Eq. (2), we have developed and fabricated transmission-type twisted-nematic liquid-crystal logic elements and a spatial light modulator (SLM). The unit-cell structure of the liquid-crystal devices is shown in **Fig. 1**. Liquid-crystal molecules have a 90° twisted alignment between the two parallel indium tin oxide (ITO) electrodes.[7] In the off-state (no voltage to the electrode) shown at (a) in **Fig. 1**, the direction of polarization of linearly polarized incident light is rotated by 90° (exactly) through the twisted angle. Thus, the light is blocked by the analyzer parallel to the polarizer. In the on-state (voltage to the electrode) in (b), the twisted configuration is destroyed and light transmission can occur. The former provides 0 and the latter 1 in binary. The multiplications between the magnitude bits (AND logic) can be performed by a serial connection of two unit cells (as shown at (a) in **Fig. 2**). Sign multiplication operations (exclusive OR logic) can be performed by a similar serial connection of two unit cells in which the analyzer in the first unit cell and the polarizer in the second unit cell are removed and the analyzer in the second unit cell is rotated perpendicularly to the polarizer in the first (as
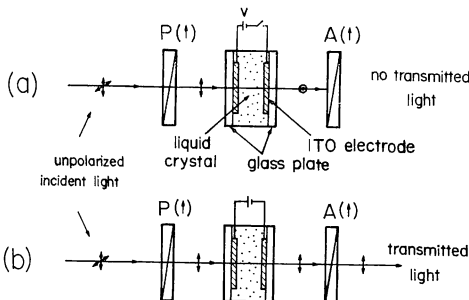

Fig. 1 Basic structure of the unit cell. (a) off-state, 0 in binary number. (b) on-state, 1 in binary number. Polarizer (P) and analyzer (A) are parallel to each other. $\updownarrow$ and $\odot$ are the polarization states of light.
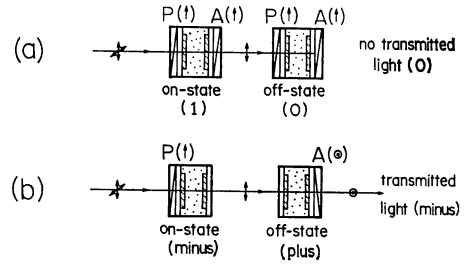

Fig. 2 (a) AND logic operation for multiplying magnitude bits ($1 \times 0 = 0$). (b) exclusive OR logic for sign bits ($1 \oplus 0 = 1$) where P and A are crossed.
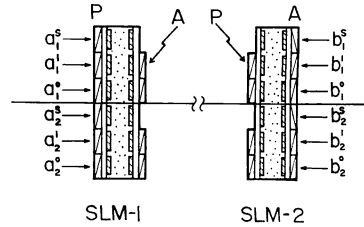

Fig. 3 Schematic architecture of transmission-type liquid-crystal SLM's. Analyzers for b_1^s and b_2^s at SLM-2 are perpendicular to the other P's and A's.

shown in (b) of **Fig. 2**). Two logic operations of the AND and exclusive OR for two binary numbers (1 and 0) are explained in **Fig. 2**. Thus, matrix C in Eq. (2) can be obtained by constructing two transmission-type liquid-crystal SLM's (**Fig. 3**), each SLM being composed of two groups of pixels. Each of these comprise one sign pixel and a set of magnitude pixels (2 pixcels in **Fig. 3**). In the case of the M-bit binary sign-magnitude representation, there are (M-1) subgroups for magnitude bits per one sign pixel.

3. Results and Discussion

In an experiment, we studied the outer product of two 2×2 matrices, each of their matrix elements being converted to an 8-bit binary word, including a sign bit. As a practical example, we show the follwing matrix product:

$$C = A \cdot B = \begin{pmatrix} -54 & -38 \\ 23 & 63 \end{pmatrix} \begin{pmatrix} 58 & -29 \\ -46 & 59 \end{pmatrix}. \quad (3)$$

The over-all optical computing system is shown in **Fig. 4**. Two liquid-crystal SLM's are crossed and inputs to the SLM's are made by an interfaced 6502 micro-processor. Two outer product matrices, C_1 and C_2, resulted from multiplying the column vectors of A with the row vectors of
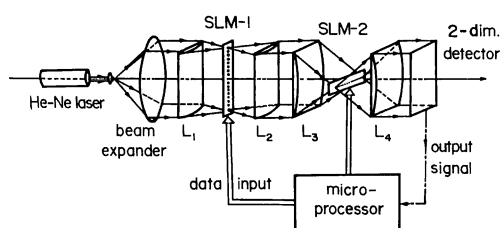
Fig. 4 Experimental arrangement for optical matrix multiplication. SLM-1 (two words, one sign pixel and 7 magnitude pixels per word) and SLM-2 are the liquid-crystal spatial light modulators and L_i ($i=1,2,3$ and 4) are cylindrical lenses.

B (obtained at a 2-dimensional detector plane). C_1 is given as

$$C_1 = \begin{pmatrix} -54 \\ 23 \end{pmatrix} (58 \quad -29)$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} (00111010 \quad 10011101) = \begin{pmatrix} 1 & \vdots & 0 \\ 0000000 & 0000000 \\ 0111010 & 0011101 \\ 0111010 & 0011101 \\ 0000000 & 0000000 \\ 0111010 & 0011101 \\ 0111010 & 0011101 \\ 0000000 & 0000000 \\ \cdots & \cdots \\ 0 & \vdots & 1 \\ 0000000 & 0000000 \\ 0000000 & 0000000 \\ 0111010 & 0011101 \\ 0000000 & 0000000 \\ 0111010 & 0011101 \\ 0111010 & 0011101 \\ 0111010 & 0011101 \end{pmatrix} .$$

(4)

The bits resulting from multiplying the sign and magnitude bits were removed for clarity (as in Eq. (2)). C_2 has a similar matrix representation. The experimental results of the outer product matrices, C_1 and C_2, are shown in **Fig. 5**. The bright spots mean 1's and the dark 0's in binary. Also, the dark or bright spot at the left-top corner of each submatrix gives plus or minus signs, respectively. The other spots give the magnitudes of the elements. These elements can be represented in decimal notation via summations of the cross-diagonal terms and weighted summations of mixed binary numbers.[6] This is illustrated for C_1 in Eq. (4) and C_2 as follows :

$$C_1 = \begin{pmatrix} 10012223311100 & 00001222331110 \\ 00001122422110 & 10000112242211 \end{pmatrix} \quad (5)$$
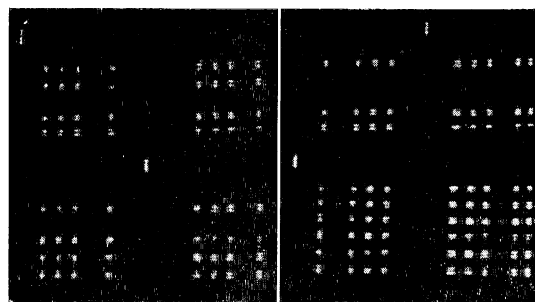
（a）C_1　　　　　　（b）C_2

Fig. 5 Experimental results for the outer product matrices C_1 of Eq. (4) and C_2.

$$= \begin{pmatrix} -3132 & 1566 \\ 1334 & -667 \end{pmatrix} \quad (6)$$

and

$$C_2 = \begin{pmatrix} 1748 & -2146 \\ -2898 & 3717 \end{pmatrix} . \quad (7)$$

Therefore,

$$C = C_1 + C_2 = \begin{pmatrix} -1384 & -580 \\ -1564 & 3050 \end{pmatrix} . \quad (8)$$

4.　Conclusion

We have developed transmission-type twisted nematic liquid-crystal SLM's for performing logic operations and the matrix multiplication of a binary outer product decomposition. The present processor offers both positive and negative element handling without the conversion of bipolar data into unipolar data or the matrix partitioning when the matrices of interest are real. The input of both positive or negative matrix elements to the SLM's in the binary representations and the decoding of the outer product matrices into decimal representations could be accomplished by using a micro-processor. The optical architecture can be extended to a triple product processor by using another 1-dimensional liquid crystal SLM.[8] The liquid crystal SLM is reasonably easy to prepare. Owing to its compactness, solid-state construction, low cost and low-voltage requirements ($\sim$5 V), it is also easy to use.[9] The response times, which are of the order of 50 ms for switch-on (from zero to maximum transmitted light intensity) and 100 ms for switch-off (from maximum to zero intensity), are inherently slow. We also look forward to progress in the improvement in the response time of liquid crystals to be of the order of a microsecond.[10]

Acknowledgement

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