研 究

Coherent Ladar Equation and Radar Equation

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The coherent ladar equation is given and described in the paper, and compared to the radar equation. Both equations have the same form. The coherent mixing factor of the coherent ladar equation, which is important in ladar design, is discussed.

1. Introduction

Over the last 20 years, coherent laser radars have developed rapidly. During this period some studies were carried out from theory to the test system. In the past 10 years, CO_2 coherent laser radars have developed rapidly; many institutes have established experimental systems and some of them advanced to field tests. All of these developments indicate that the design of coherent laser radar systems has become an important study topic. In the design of a laser radar system, an important parameter of interest is the range of the radar. The range of a conventional radar is determined by its range equation, which is deduced from the propagation theorem of the plane electromagnetic wave. But for the coherent laser radar, the radar equation must be derived from the extended Huygens-Fresnel principle.[1] Furthermore, it should be noted that for conventional radars some targets can be regarded as glint targets which may not be so for the coherent laser radar. Hence, in general, the coherent laser radar equation must treat glint and speckle effects of a target[2] as well as the shapes of the signal beam and the LO beam. These points are different from conventional radars.

In this paper, we discuss the received signal power for some receiver forms and the range equation from of coherent laser radars (ladars), and compare them with the conventional radar equations. A coherent mixing factor of the ladar is discussed and calculated numerically for some wavefront shapes in the equation. In the discussion in this paper, the signal beam and the LO beam are assumed to be aligned perfectly, the distribution of the quantum efficiency on the detector surface is uniform, the turbulence effect of the atmosphere on optical beams is not considered, and the transmitting beam is irradiated over the entire target.

2. Radar Equation and Ladar Equation

In general, the transmitting beam of conventional radar is much broader than the target; thus, it can be considered as a plane wave beam. According to the propagation theorem of the plane electromagnetic spectrum, the conventional radar range equation is written as[3]

$$P_r=\frac{P_t}{4\pi r^2}\times G_t\times\frac{\sigma}{4\pi r^2}\times A_r\times\eta_{atm}\times\eta_{sys}, \quad (1)$$

where P_r is the received power, P_t is the transmitted power, r is the range, G_t is the antenna gain, σ is the radar cross section of the target, A_r is the effective area of the receiver antenna, η_{atm} is the atmospheric transmission factor, and η_{sys} is the system transmission factor.

The antenna gain is the ratio of the actual power density delivered by the system at the object plan to the expected power density delivered by a hypothetical isotropic (uniformly radiating) antenna. The radar cross section is that in which the effective area of a target produces the reflection of the target. Thus, the received power is a direct function of transmitter power, antenna gain, the effective target cross section, the area of the receiver antenna, and the atmospheric and system transmission factors.

For comparison with the radar range equation, the ladar range equation is written as[4]

$$P_r=\frac{P_r}{4\pi r^2}\times G_t\times\frac{\sigma}{\Omega_r r^2}\times A_r\times\eta_{atm}\times\eta_{sys}, \quad (2)$$

where Ω_r is the solid angle of the beam dispersed by the target. Other terms are the same as radar equation (1). Note that the form of equation (2) is similar to that of equation (1), except for the term of Ω_r in equation (2), which is replaced by 4π in equation (1). As the laser beam has good direction, in general, the target reflects the signal beam at a certain solid angle (Ω_r). For the conventional radar range equation a target is considered to reflect the signal field in all directions (4π). Therefore it can be said that both equations have the same form.

3. Coherent Ladar Equation

Since the two radar equations described above are suitable for directly receiving forms, they are unrelated to the wavefront of the transmitting beam. For laser radars, another receiving form is heterodyne reception. **Figure 1** depicts a schematic of a heterodyne reception laser radar system. The returned signal, along with the local oscillator (LO), is incident on a detector. In general, the propagation of the signal beam through the atmosphere is described by the Huygens-Fresnel principle model. If the atmosphere turbulence is not considered, the receiver signal power of heterodyne detection, in terms of the principle, is as given by Wang.[5] From the results of several practical wavefront cases, we find that the received signal powers P_r have a common term, i. e., for a glint target,

$$\frac{P_t T_g(\Delta\rho)^4 \beta_1{}^4}{(4d_1)^4}\left(\frac{d_2}{d_1}\right)^2.$$

To coincide with the conventional radar equation, we make some substitutions. The transformed result is

$$P_r=\frac{P_t}{4\pi r^2}\times G_t\times\frac{\sigma}{\Omega_r r^2}\times A_r\times\eta_{atm}\times\eta_{sys}\times\eta_m, \tag{3}$$

where

$$G_t=\frac{4\pi}{\Omega_t};\ \sigma=T_g A_t;\ \Omega_t=\frac{\pi}{4}\left(\frac{4}{\pi}\frac{\lambda}{d_1}\right)^2;$$

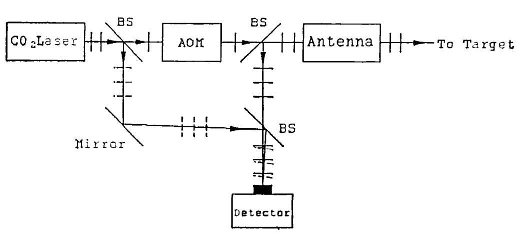

Fig. 1 A schematic of heterodyne receiving system for uniform signal and uniform LO.

$$\Omega_r=\frac{\pi}{4}\left(\frac{4}{\pi}\frac{\lambda}{d_t}\right)^2;\ A_t=\frac{1}{4}\pi d_t{}^2;\ A_r=\frac{1}{4}\pi d_2{}^2.$$

Here, A_t is the equivalent area of the single glint target, Ω_t is the solid angle of the transmitter antenna, Ω_r is the retroreflective solid angle of the cooperated (or glint) target, d_1 and d_2 are efficient diameters of the transmitter and receiver antennas, respectively, and other symbols are the same as for equation (1). Here a factor η_m, coherent mixing efficiency, is introduced. After deduction and combination, η_m can be divided into two parts. For a common aperture system, η_m can be written as

$$\eta_m=I_s{}^2\times I_L{}^2, \tag{4}$$

where I_s and I_L are related to the wavefront shapes of the signal beam and the LO beam, respectively. Hence η_m reflects the wavefront matching level; for a uniform signal and the LO beam, it is one and meets the requirement of the normalization. For some wavefront shapes, the I_s and I_L are given as follows:

(a) Uniform wavefront

$$I=\frac{\sin(\beta/8)}{\beta/8}, \tag{5}$$

(b) Gaussian wavefront

$$I=\frac{32\xi^3}{16\xi^4+\beta^2}1-\sqrt{1+\left(\frac{\beta}{4\xi^2}\right)^2}\exp\left(-\frac{\xi^2}{2}\right)\times\cos\left[\frac{\beta}{8}+\tan^{-1}\left(\frac{\beta}{4\xi^2}\right)\right], \tag{6}$$

(c) Airy wavefront

$$I=4\int_0^{1/2}\mathrm{d}r J_1(Qr)\exp\left(\frac{j}{2}\beta r^2\right), \tag{7}$$

In the above formulas,

$$\beta=\frac{kd^2}{r};\ \xi=\frac{d}{2\omega};\ Q=\frac{kdd_0}{2f};$$

where β is Fresnel number, 2ω is the diameter of the beam waist, ξ is the truncation ratio of the transmitter beam or the LO beam, Q is a parameter related to the detection efficiency, and its optimum value equals 5.5 if the signal and LO beams are aligned.[6] $J_1(x)$ is a Bessel function of the first order.

For a speckle target, the common term of the received signal powers P_r can be written in the form

$$P_t\beta_1{}^4\left(\frac{T_d}{\pi}\right)\left(\frac{\lambda}{d_1}\right)^2\left(\frac{d_2}{d_1}\right)^4\left(\frac{d_t}{d_2}\right)^2\frac{32}{\pi}.$$

To coincide with the conventional radar equation, we make some substitutions, and the transformed result is given by

$$P_r=\frac{P_t}{4\pi r^2}\times G_t\times\frac{\sigma}{4\pi r^2}\times A_r\times\eta_{atm}\times\eta_{sys}\times\eta_m, \tag{8}$$

where the meanings of the introduced symbols are the same as those for the glint target, except that the glint reflection coefficient T_g is replaced by the mean target reflectivity T_d. Here η_m is also the coherent mixing efficiency. For a common aperture system, it becomes

$$\eta_m = 8\int_0^{1/2} \rho \, d\rho I_s^2 I_L^2, \quad (9)$$

where I_s and I_L are related to the wavefront shape of the signal beam and the LO beam, respectively. For various wavefront shapes, I_s and I_L are given as follows:

(a) Uniform wavefront

$$I = 8\int_0^{1/2} d r r J_0(\beta\rho D r) \exp\left(\frac{j}{2}\beta r^2\right) \quad (10)$$

(b) Gaussian wavefront

$$I = 8\xi\int_0^{1/2} d r r J_0(\beta\rho D r) \exp\left[-2\xi^2 r^2 + \frac{j}{2}(\beta-\alpha) r^2\right] \quad (11)$$

(c) Airy wavefront

$$I = 4\int_0^{1/2} d r J_1(Q r) J_0(\beta\rho D r) \exp\left(\frac{j}{2}\beta r^2\right). \quad (12)$$

In the above formulas,

$$D = \frac{d_T}{d}; \; \alpha = \frac{kd^2}{F},$$

where D is the ratio of the target diameter d_T to the diameter d of the common antenna. $J_0(x)$ is a Bessel function of the zeroth order. The focus F of the optical antenna telescope is infinite, in general. Therefore, α can be considered to be zero.

From the above results, we can conclude that the equation of the glint target is similar to the ladar equation, and the speckle equation has a form similar to the conventional radar equation except for the addition of coherent mixing efficiency. Equations (3) and (8) can be called the coherent ladar equations, and the coherent mixing efficiency η_m reflects the degree of matching between the signal and the LO beams. The efficiency curves of the various matching curves for the glint target and speckle target are computed numerically, as shown in **Fig. 2** and **Fig. 3**. Considering the range of ladars and the laser beam divergence angle, the size of the light spot on the target is of the same order as the diameter of the antenna. Therefore we choose the ratio D of one in Fig. 3. In the case of the far field ($\beta=0$), the matching of the LO and the signal is almost complete (because various phase fronts in this case can be considered as plane wavefronts relative to the receiving aperture); hence, the efficiency is very high and approximately one. For the glint target, when β is less than about 10, the efficiency values for the uniform-uniform, the Gaussian-uniform and the Gaussian-Gaussian forms decrease in order, but with $\beta > 10$, the efficiency tends to be the same, and for the Gaussian-Airy form, it is always low. At the same time, the efficiency characteristic for the speckle target is similar to that of the glint target, but for greater D the curves change markedly toward larger β values. From these figures, the coherent ladar parameters in system design should be chosen such that the value of β lies in the middle field. If β is in the far field, η_m is very high, but the range r is long, and the receiving power is low still. If β is in the near field, η_m is very small and the power is also insignificant.

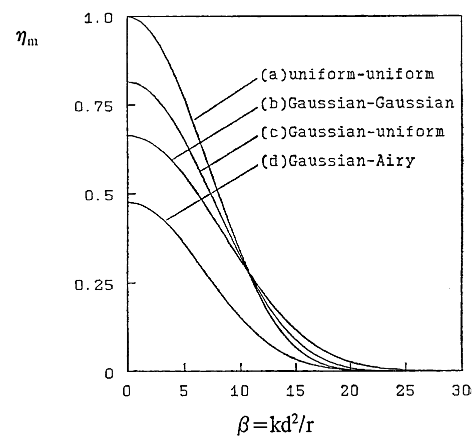

Fig. 2 The coherent mixing efficiencies of various matching forms for a single glint target.

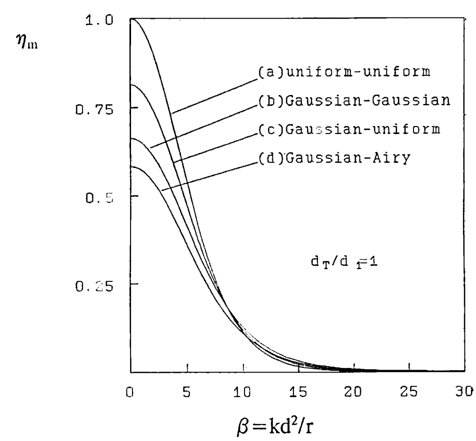

Fig. 3 The coherent mixing efficiencies of various matching forms for a speckle target.

4. Radar Equation and Ladar Equation

All radars and ladars consist of similar functional elements, so similar characteristics should be exhibited. One of the common features is the equation describing their receiving power, which can be expressed as

$$P_r = \frac{P_t}{4\pi r^2} \times G_t \times \frac{\sigma}{\Omega_r r^2} \times A_r \times \eta_{atm} \times \eta_{sys} \times \eta_m, \tag{13}$$

where the meanings of these symbols are the same as stated above. Here, Ω_r is the solid angle of the retrodirective reflection of cooperated targets. For the noncooperated targets, Ω_r equals 4π. For directly receiving ladar and radar, the coherent efficiency η_m is not considered and is replaced by unity. For a coherent reception, the efficiency is often less than 1, which is described in the above discussions. Here we have obtained an equation describing the receiving power of coherent ladars, and its form is the same as that of conventional radars and ladar, indicating the common property between radar and ladar.

5. Conclusions

The range equations of radar, ladar and coherent ladar are discussed and compared. A common form is deduced which shows the common characteristic; however, differences exist between them. For the coherent ladar equation, the coherent mixing factor is an important parameter in ladar design, so the coherent mixing efficiencies of various matching conditions have been calculated numerically. In coherent ladar design, the choice of matching forms makes a trade-off with efficiency.

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