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Wavelet versus Fourier Transforming: A Comparative Study on Electrochemical Aggregates

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1. Introduction

In this paper, we want to compare two optical methods to study fractality, namely *diffraction* and the *optical wavelet transform* (OWT). We will focus on the particular case of a physical 2-dimensional aggregate to show how different aspects of fractality are revealed by these two methods.

A number of mathematical objects are rigorously defined as fractals in the context of specific recursive construction rules.[1] Mass scaling,

$$M(\epsilon, \vec{x}) \sim \epsilon^{\alpha(\vec{x})} \qquad (1)$$

is a basic property of these objects. Here M is the mass of the object inside a box of size ϵ, around a point $\vec{x}$ of the fractal, and $\alpha(\vec{x})$ is a "local pointwise dimension." If α is constant everywhere, the fractal is said to be globally self-similar. If α takes on different values, the structure is said to be multifractal. The fractal dimensions D_q's are defined from the set of α values.[2] Self-similar structures have a unique fractal dimension: $\alpha = D_q = \text{cst}\ \forall q$.

The values of α and those of D_q can be deduced from a wavelet transform (WT) of the fractal mass density, $\rho(\vec{x})$. Very briefly, the WT amounts to convoluting ρ with a "wavelet" of adjustable size, a. Most interestingly, this can be done optically with coherent light through a series of bandpass filterings.[3] A complete OWT set-up has been developped in our laboratory and has been operated routinely for two years.

In the physical world, growth phenomena often produce highly ramified and apparently disordered structures, whose mass distribution scales according to (1) and thus are termed "fractals." Clusters grown by diffusion limited aggregation (DLA) are the most popular example of such structures.[4] Using the OWT technique, we checked that the DLA clusters are self-similar, with $D_q \simeq 1.6\ \forall q$.[5]

In general, the spectral density $S(\vec{k})$ ($\vec{k}$ is the spatial frequency) of physical fractals is expected to scale according to:

$$S(\vec{k}) \sim |\vec{k}|^{-D_2} \qquad (2)$$

Therefore, diffraction is an efficient way to evidence fractality. But notice that (2) gives only one dimension—the so called "correlation dimension"—and thus cannot resolve multifractality.

Figure 1 shows a 2-dimensional metallic cluster which was grown from a metallic solution by electrodeposition.[6] In conditions of very low voltage and low concentration, the deposition is controlled mostly by the diffusion of ions and the resulting structure approaches that of an ideal DLA cluster. As we will see, the analogy is only approximate. In the following, we will take this illustrative example to compare the capabilities of diffraction and OWT in the study of fractal growth.

2. Spectral Density

Figure 2 (a and b) shows the spectral density of the electrodeposition aggregate (EA) of Fig. 1, together with that of a numerical DLA cluster of roughly the same size (Fig. 2 c). Curve a) was obtained by Fraunhofer diffraction through a photographic slide representing the EA. Curve b) is the result of a numerical Fourier transform (FFT). We chose $k_g = 2\pi/R_g$ as the unit spatial frequency. Here R_g is the gyration radius of the aggregate.

Looking at a) and b) we notice three regimes:

1. the similarity of the EA to a DLA cluster, i. e mass fractality, is true only in a restricted spatial frequency range, $3 \leq k/k_g \leq 22$ (region II in Fig. 2). The corresponding correlation dimension D_2 is about 1.7.
2. at high frequencies, $k/k_g > 22$ (region III in Fig. 2), the EA spectral density notably deviates from that of the DLA cluster. The slope of the log-log plot in this region is about 2.4, a value higher than the space dimension ($d=2$).
3. at small frequencies, $k/k_g < 3$ (region I in Fig. 2), $S(k)$ depends very slowly on k.

These high and low frequency regimes can be explained most simply by the following considerations:

- High frequencies correspond to scales smaller than about $R_g/22$. In this regime, the cluster structure of the EA is mainly that of a massive object bounded by a highly corrugated frontier, in fact a "contour fractal." If D is the contour dimension ($1 \leq D \leq 2$), we expect[7]
$$S(k) \sim k^{D-4}, \qquad (3)$$
i. e a slope larger than 2 in a log-log plot. This indeed fits to the large frequency behavior in Fig. 2 a and b.
- Low frequencies correspond to scales larger than $R_g/3$. The behavior of $S(k)$ in this range suggests that fractality is lost at large scales in EA. To illustrate this point, we broke an

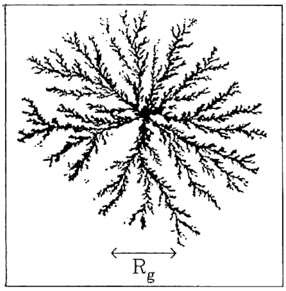

Fig. 1

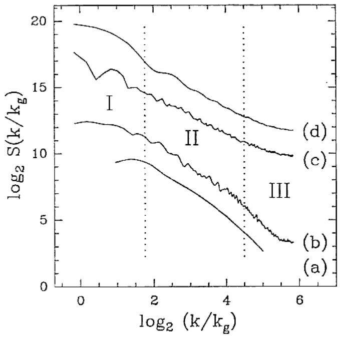

Fig. 2

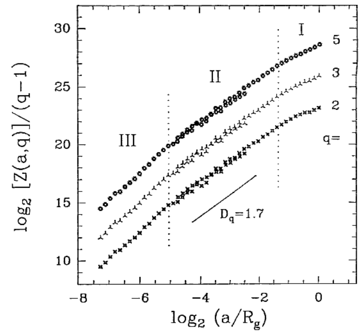

Fig. 3

Fig. 1 A cluster grown by electrodeposition. R_g is the gyration radius of the cluster.
Fig. 2 The spectral density of an electrodeposition aggregate compared to that of a DLA cluster in a $\log_2$-$\log_2$ representation. See text for the definitions of the different graphs.
Fig. 3 The partition function of the mass distribution in the electrodeposition aggregate, built from its OWT, versus the wavelet size, in a $\log_2$-$\log_2$ representation.

ideal DLA cluster (the one from which curve c) was computed) into pieces of about $R_g/5$ in size; then we calculated the spectral densities of the different pieces and added them uncoherently. This gave curve d), which very well mimics the behavior of the EA cluster at large scales. This definitely suggests that the fractal structural hierarchy does not persist up to large scales in the electrodeposition growth.

3. Optical Wavelet Transform

Figure 3 shows the behavior of the partition function[5)]

$$Z(a, q)=\int d^2x\rho(\vec{x})|a^2\sqrt{I(a, \vec{x})}|^{q-1} \qquad (4)$$

as a function of the reduced size a/R_g, in a log-log representation. Here $I(a, \vec{x})$ is the intensity of the optical wavelet component of the sample for a wavelet of size a. In the mass fractal regime, we expect[5)]:

$$Z(a, q)\sim a^{(q-1)D_q} \qquad (5)$$

This regime is clearly visible in Fig. 3 in a scale range that corresponds to the intermediate frequency range identified in Fig. 2 (region II in Fig. 2). In addition to D_2 the OWT gives the whole set of D_q's dimensions. All the slopes in the mass fractal regime in Fig. 2 are found equal to about 1.7. This proves the global self-similarity of EA in this regime, as expected from the analogy with DLA growth.

At large scales, the slopes in Fig. 3 decrease down to values slightly higher than 1 (region I in Fig. 3). The behavior of Z in this range is different from that of the spectral density, but there is no contradiction. The analogy we made in paragraph 2 with the pulverized DLA cluster is equivalent to considering that, at large scales, the mass density of EA looks like a white noise. We calculated $Z(a, 3)$ and $Z(a, 5)$ in the limit of a white noise and found that they behave as a^2 and a^4, respectively. This corresponds to a slope equal to 1 in the representation chosen in Fig. 3. With this view, the large scales in Fig. 3 appear to be in a cross-over regime between the mass fractal behavior and the white noise (random cluster) limit.

The behavior of the WT of contour fractals has not been explored up to now. We just made an estimation in the particular case of a domain bounded by a Koch curve.[1)] In fact, we found that $Z(a, 2)$ and $Z(a, 3)$ do scale according to (5), with an apparent dimension slightly larger than 2. Thus, scales lower than $R_g/22$ in region III in Fig. 3 appear to be in the contour fractal regime, in agreement with the estimation from the spectral density (Fig. 2).

In conclusion, we hope we have conveyed some taste of the specific possibilities given by the optical wavelet transform to study mass scaling in fractals. We believe that the example of the electrodeposition aggregate is a good illustration of what the wavelet transform of a mass distribution looks like, from a massive object to an ideal gas, through the contour fractal and mass fractal regimes.

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