研究速報

Optical Implementation of Holographic Associative Memory Based on the Symmetric Three-Layered Network

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We have developed a new optical associative memory system based on the symmetric three-layered neural network model. It is optically implemented using scattered light holograms and an LCTV spatial light modulator. In the experiment and the simulation, four Korean characters are used as memory patterns. The simulation results are compared with those of the Hopfield model, which show that more than 95% recognition probability is obtained for the inputs within the error rate of 12%.

1. Introduction

Major properties of neural networks are massive parallelism and information storage in distributed manners by interconnection of processing units called neurons. Optical information processing using holograms and optoelectric devices inherently has such properties. Optical implementations of Hopfield model[1] and various holographic associative memories (HAMs) have been developed by many authors.[2-12] Recently, Caulfield[3] suggested a method of HAM storing several objects by multiple exposures with a diffuser, and Song and Lee[8] optically implemented and called it scattered light holographic associative memory (SL-HAM). In an associative memory such as the Hopfield model, the interconnection matrix is obtained by adding all the outer products of memory vectors (or patterns). It has the advantage that implementation can be done easily with vector-matrix multiplication devices or holographic systems, but the limitations of the storage capacity caused by the existence of spurious states, unwanted local minima, and oscillating states are drawbacks of Hopfield-type models. In order to increase the storage capacity, some authors have introduced and optically implemented quadratic associative memories (QAMs),[13-15] associative memory utilizing nonlinearity in the correlation domain,[16,17] and adaptive learning system which learns dynamically the interconnection (synaptic) weight through iterative adaptation.[18-21] In this paper, we experimentally demonstrate and discuss a new HAM based on the symmetric three-layered network (STLN). We named it STLN because the input and the output layer are the same as that of two-layered associative memory except intermediate (middle) layer. It is implemented by SL-HAM and a liquid-crystal television (LCTV). Simulation results are compared with those of the Hopfield model. Finally, we also discussed the limitations of the storage capacity theoretically.

2. Principle of STLN

The principle of the symmetric three-layered (input, intermediate, output) associative memory using outer-product storage is summarized as follows. U^m $(m=1, 2, \cdots, M)$ is mth memory vector with N bit binary (1, 0) elements, and it corresponds to the input and output units. G^m is associated with U^m, and it corresponds to the intermediate units. W^1 is an interconnection matrix connecting input units U^m to intermediate units G^m as shown in **Fig. 1**, and it is obtained by outer-product operation between U^m and G^m.

$$W^1 = \sum_{m=1}^{M} G^m (U^m)^T, \qquad (1)$$

where the superscript T represents the transpose operation to make a column vector into a low vector, and *vice versa*. W^2 is an interconnection matrix connecting intermediate units G^m to output

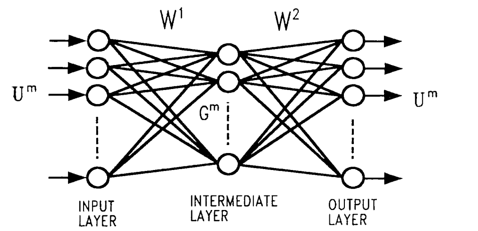

Fig. 1 Schematic diagram of symmetric three-layered network model.

units U^m and it has a transpose relation with W^1.

$$W^2=\sum_{m=1}^{M} U^m(G^m)^T. \tag{2}$$

Let G^m be an orthogonal vector having M, which is the number of memory vectors, binary elements as follows:

$$G_j^m=\begin{cases}1, & \text{if } j=m,\\ 0, & \text{otherwise,}\end{cases} \tag{3}$$

where G_j^m is jth element of mth vector G^m. Then the inner product between G^m and G^n can be written as follows:

$$(G^m)^T G^n=\delta_{mn}, \tag{4}$$

where δ_{mn} is a Kronecker delta. The retrieval of stored information from input U^k can be represented by matrix-vector multiplication with nonlinear thresholding operation. When the interconnection matrix W^1 is multiplied by the input U^k, then G^{out} is obtained as follows:

$$\begin{aligned}G^{\text{out}}&=W^1U^k,\\ &=\sum_{m=1}^{M}[G^m(U^m)^T]U^k,\\ &=\sum_{m=1}^{M}G^m[(U^m)^TU^k],\\ &=\sum_{m=1}^{M}G^m\alpha(m,k),\\ &=[\alpha(1,k),\alpha(2,k),\cdots,\alpha(M,k)],\end{aligned} \tag{5}$$

where $\alpha(m,k)$ is an inner product between mth memory vector U^m and the input vector U^k. Because G^m has only one non-zero mth element, all inner product values $\alpha(m,k)$ are spatially separated in the intermediate layer. In two-layered network, however, all inner product values are accumulated in the output plane. If the input U^k is the most close to the sth memory vector U^s, then G^s is obtained as intermediate output by winner-takes-all (WTA) operation $f[\cdot]$ which is a nonlinear thresholding operation choosing only a maximum value.

$$\begin{aligned}G^s&=f[G^{\text{out}}],\\ &=f[\alpha(1,k),\cdots,\alpha(s,k),\cdots,\alpha(M,k)],\\ &=(0,\cdots,1,\cdots,0)^T.\end{aligned} \tag{6}$$

Finally, U^s is obtained as an output when W^2 is multiplied by G^s.

$$\begin{aligned}U^{\text{out}}&=W^2G^s,\\ &=\sum_{m=1}^{M}U^m(G^m)^TG^s,\\ &=\sum_{m=1}^{M}U^m\delta_{ms},\\ &=U^s.\end{aligned} \tag{7}$$

Therefore, we obtain an output which is the same or most similar to the input vector U^k.

3. Principle of Outer-Product Implementation

Consider, for simplicity, one-dimensional memory patterns with 5 bit binary elements:

$$b=(b_1, b_2, b_3, b_4, b_5)^T. \tag{8}$$

Figure 2(a) shows the diagram of constructing the interconnection matrix by the outer-product storage holographically. The collimated beam from the plane I_1 and the coherent scattered beam from the plane I_2 with a diffuser illuminate the holographic plate, and they construct the interference fringes. The element of the interconnection matrix B_{ij} represents the interference state, anb it is given by

$$B_{ij}=b_ib_j. \tag{9}$$

If b_j (jth pixel of the pattern on I_1) and b_i (ith pixel of the pattern on I_2) are both transparent, they make interference pattern ($B_{ij}=1$). However,

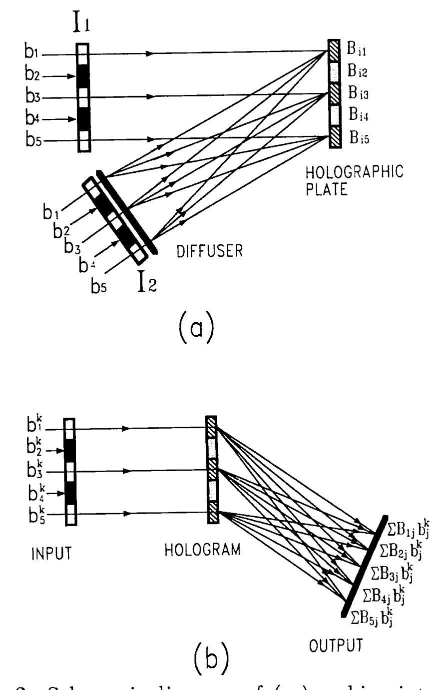

Fig. 2 Schematic diagram of (a) making interconnection matrix holographically and (b) recognizing the input b^k.

if one of two pixels or both are opaque, then they do not make interference pattern ($B_{ij}=0$). The first column of the matrix B_{i1} correspond to the interference patterns between the beam from the transparent pixel b^1 of I_1 and all scattered beams from b_i ($i=1, 2, \cdots, 5$) of I_2. Even though B_{i1} is overlapped on the same place of the hologram, they are distinguished by their different phase informations due to the different scattering angles of beams from I_2. Figure 2(b) shows the diagram of reconstruction of the stored memory pattern by illumination of the input b^k on the hologram. The ith component of output is the summation of diffracting beams from the hologram to the ith position, and it is proportional to b_i which is the ith component of the stored pattern.

$$\begin{aligned} b_i^{out} &= \sum_{j=1}^{5} B_{ij} b_j^k, \\ &= B_{i1}b_1^k + B_{i2}b_2^k + B_{i3}b_3^k + B_{i4}b_4^k + B_{i5}b_5^k, \quad (10) \\ &= b_i\left(\sum_{i=1}^{5} b_i b_i^k\right). \end{aligned}$$

If several memory patterns are stored by multiple exposures, unwanted noise patterns due to the cross-correlation also appeared at the output plane, then threshold operation is needed to get a noiseless output. This principle of SL-HAM can be applied directly to the two-dimensional patterns.

4. Experiment and Simulation Results

In the experiment, we take up objects of 2-D Korean characters (Hangul) as four memory patterns. They are constructed with 25(5×5) binary (transparent/opaque) pixels (2 mm×2 mm) as shown in **Fig. 3**(a) and represented as vector forms of $M=4$, $N=25$ by ordering the pixels in a row:

$$\begin{aligned} U^1 &= (0111000001001110010000111 1)^T, \\ U^2 &= (0110001100011001011010011)^T, \\ U^3 &= (0111010001100011000101110)^T, \\ U^4 &= (1111100100010101000110001)^T. \end{aligned} \quad (11)$$

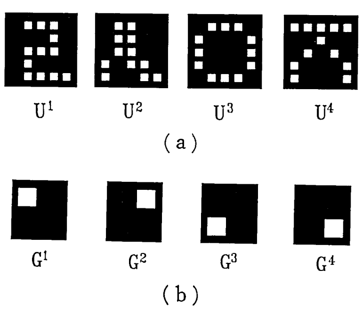

Fig. 3 (a) Four memory patterns and (b) four orthogonal intermediate patterns.

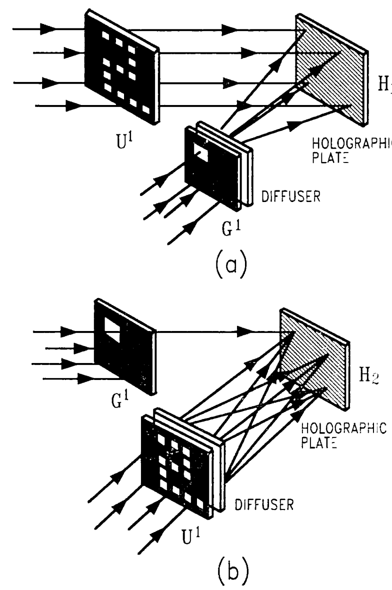

Fig. 4 Schematic diagram of making (a) the first and (b) the second hologram.

We note that all memory vectors consisting of 25 elements with the same numbers of twelve 1's and thirteen 0's. Four orthogonal intermediate patterns associated with those memory patterns are shown in Fig. 3(b), and they are written as follows:

$$\begin{aligned} G^1 &= (1000)^T, \\ G^2 &= (0100)^T, \\ G^3 &= (0010)^T, \\ G^4 &= (0001)^T. \end{aligned} \quad (12)$$

The schematic diagram of making the first hologram (H_1), corresponding to the interconnection matrix W_1 in Eq. (1), is shown in **Fig. 4**(a). The interference fringes, formed by the collimated beam passing through the pattern U^1 and the scattered beam from the diffuser placed behind the pattern G^1, constructs holographically the vector outer-product between U^1 and G^1 on the recording medium (Kodak HRP type 1A). The holographic recording medium is exposed four times successively to superimpose all interference fringes between U^m and G^m ($m=1, 2, 3$ and 4). Figure 4(b) represents the schematic diagram of making the second hologram (H_2) corresponding to the interconnection matrix W_2 in Eq. (2). The principle of constructing the outer-poduct holographically is the same as that of H_1. Instead the patterns U^m and G^m should be exchanged so that H_2 can have transpose relation with H_1 as shown in Eq. (1) and Eq.(2). The diffuser is made of a flat optical glass ground with silicon carbide powder of grain size 80-100 μm.

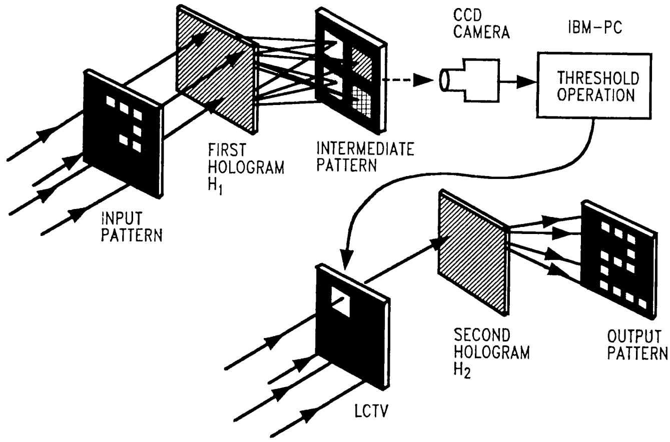

Fig. 5 Experimental setup of reconstruction process.

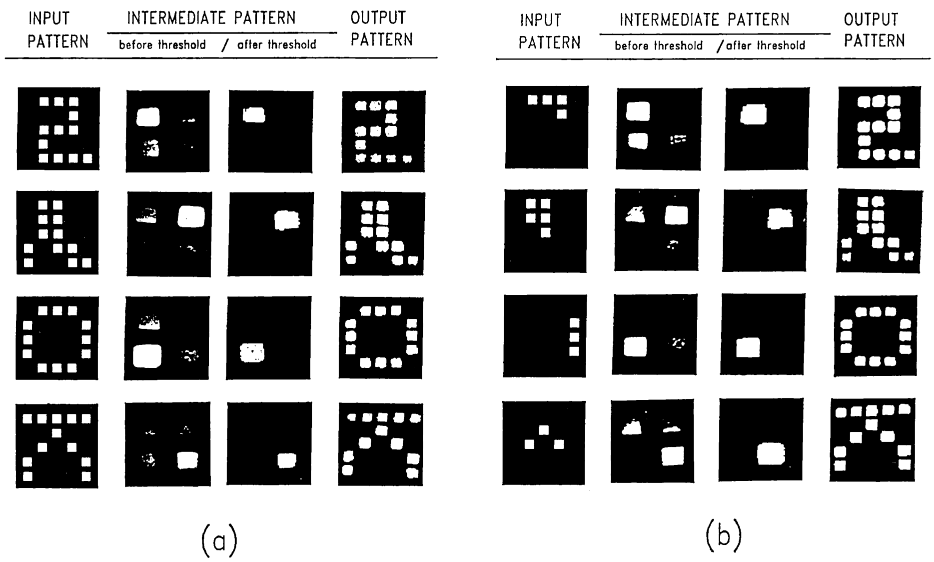

Fig. 6 Experimental results of recognizing the various inputs as (a) memory patterns and (b) distorted (partial) patterns.

Figure 5 shows the optical system for recognizing a partial or distorted input of the stored memory pattern. The first order diffracted beam from H_1 gives the intermediate pattern (before threshold). A CCD (charge-coupled detector) camera is used to detect this image. The nonlinear threshold operation WTA is carried out by using an electronic processor connected to the CCD camera. The thresholded image, which is the intermediate pattern (after threshold), is displayed on the LCTV used as a 2-D spatial light modulator (SLM). The first order diffracted beam from H_2, placed behind the LCTV, gives the final output that is one of four stored memory patterns, and it is the closest to the input.

Figure 6(a) shows the experimental results. Intermediate patterns (before and after threshold) and output patterns are illustrated for the input patterns. We know that all memory patterns are stored stably in this system. Figure 6(b) shows the considerable error-correction capability of recognizing the partial inputs. To see the overall performance of the STLN quantitatively, computer simulations were carried out and the results were compared with those of the Hopfield model.

Figure 7 represents the results of the simulation. The recognition probability (or capability) is defined as the number of correct recognition per total number of inputs. The simulations were repeated for 500 randomly generated inputs of each error bits (Hamming distance), and the results are averaged for all memory patterns. The results show

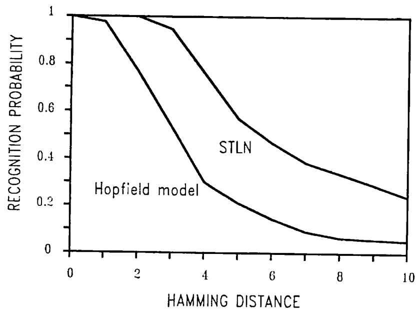

Fig. 7 Results of computer simulation of the symmetric three-layered network (STLN) and the Hopfield model.

that more than 95% recognition probability is obtained for STLN, but 54% is obtained for Hopfield model within the Hamming distance 3 (12% error rate in pattern). The storage capacity may be proportional to the signal-to-noise ratio (SNR). The interconnection matrix of the two-layered networks including the Hopfield model is given by

$$B=\sum_{m=1}^{M} b^m(b^m)^T. \tag{13}$$

When B is multiplied by the input b^k, the output b^{out} is obtained as follows:

$$\begin{aligned} b^{\mathrm{out}} &= Bb^k, \\ &= \sum_{m=1}^{M} b^m(b^m)^T b^k, \\ &= N_0 b^k + \sum_{m\neq k}^{M} b^m \alpha(m,k), \end{aligned} \tag{14}$$

where N_0 is the number of 1's of the input b^k, and $\alpha(m,k)$ is the inner product between b^m and b^k. The first term on the right-hand side of Eq.(14) is the signal, and the other is the accumulated noise. The required minimum SNR in order to recognize the input is given by

$$\begin{aligned} [\mathrm{SNR}]_{\min} &= \frac{N_0}{\left[\sum_{m\neq k}^{M} b^m \alpha(m,k)\right]_{\max}} \\ &= \frac{N_0}{\beta(M-1)} \\ &= \frac{\gamma}{M-1}, \end{aligned} \tag{15}$$

where β is the mean inner-product value given by

$$\beta=\sum_{m>n}^{M} \frac{\alpha(m,n)}{N(M-1)/2}, \tag{16}$$

$$\gamma=\frac{N_0}{\beta}. \tag{17}$$

In order to correctly recognize the input, $[\mathrm{SNR}]_{\min}$ should be larger than unity:

$$M<\gamma+1. \tag{18}$$

The mean inner-product value of four memory patterns used in this paper is $\beta=5.7$, $N_0=12$, and $\gamma=2.1$. Therefore it cannot satisfy the criterion for memory storage capacity (Eq. (18)) in the two-layered network. Because the signal and noises are spatially separated in the intermediate plane of the STLN (Eq.(5)), the storage capacity can be increased up to the number of combination of choosing N_0 out of N: ${}_N C_{N_0}$. It is about 5×10^6 for $N=25$ and $N_0=12$, theoretically. But practical capacity also depends on the resolution limit and spatial band-width product of holographic recording medium. The first hologram is divided into $25(=N)$ subholograms. Each of the subholograms has 4 $(=M)$ interference patterns superimposed on it, so that the total number of interference patterns, on the whole area (2.5 cm×2.5 cm) of the recording medium, is $100(=N\times M)$, which is less than 2500 $(=N^2\times M)$ of two-layer system, and far less than the resolution element density of the hologram recording medium.[22] The second hologram is divided into 4 subholograms. Each of four memory patterns is separately stored on each of these subholograms, so that only one pattern is obtained without noise patterns at the output plane.

5. Conclusion

In conclusion, we have demonstrated optical system and experimental results of holographic associative memory based on the symmetric three-layered network. The storage capacity and recognition ability have improved when it is compared with the conventional two-layered system and the Hopfield model. This system can be used in the areas of parallel image processing such as pattern recognition and image error correction. Multilayer adaptive learning may be implemented by replacing the recording medium and electronic processing of this system with nonlinear optical devices such as photorefractive crystals and Fabry-Perot etalon.

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