研究

Dammann Gratings for Selective Order Missing Spot Array

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We have developed an algorithm for designing the birary phase gratings of selective-order missing (SOM) spot arrays which selectively cancel or suppress unwanted diffraction orders. Principles and properties of SOM spot arrays which control the speed of the optimization algorithm are presented, and the results and the fabrication of the designed gratings are illustrated.

1. Introduction

Dammann gratings[1,2] (or binary phase gratings) are diffraction gratings of periodic phase-relief structures that generate spot arrays of uniform-intensity beams from an incoming beam using a Fourier transform lens as in **Fig. 1.** They are promising optical fan-out elements for applications in, for example, fiber-optic star couplers, free-space optical interconnections, and multiple imaging systems. There have been many studies on the design and fabrication of binary phase gratings (BPGs).[3-8] Recently, the phase gratings producing diffraction orders with arbitrary intensity have been reported.[9,10] Morrison and Walker have shown that even-numbered spot arrays could be obtained by applying the translation and reflection symmetry of the grating.[11,12] Even-numbered spot array (even-order missing spot array is a more accurate term) designs are especially useful in free-space optical logic devices such as S-SEEDs (symmetric-self-electrooptic effect devices).

In this paper, we developed a novel optimization method for obtaining the selective-order missing (SOM) spot array in which unwanted diffraction orders are selectively canceled or suppressed.[13] The SOM spot array may be applicable to the digital optical image processing system and the optical power to the arrays of devices such as smart pixels and optical logic gates. In Sec. 2 the theory of BPG is briefly reviewed. A conventional optimization rule for grating design of a conventional spot array and its modified rule for SOM spot array are presented in Sec. 3. Some properties which control the speed of the optimization algorithm are discussed and the results of simulation and fabrication are also presented in Sec. 4.

2. Theory of Binary Phase Grating

We consider the transmission function $g(x)$ of the binary phase grating as

$$g(x)=\exp\left[i\theta(x)\right], \qquad (1)$$

where $\theta(x)$ is the phase function having binary values 0 or π. When the grating is symmetric, $g(x)$ has simple properties as follows:

$$g(x)=\begin{cases} g(-x), \\ +1 \text{ or } -1. \end{cases} \qquad (2)$$

It may be noted that $g(x)$ is a one-dimensional structure. However, it can be easily extended to a two-dimensional structure, that is, $g(x, y)=g_1(x)\cdot g_2(y)$ when we consider the separable spot arrays in the x and y directions.[12] It is convenient to normalize the grating period to be 1. In the range of half-period of the grating, there are N transition points at which $g(x)$ changes its value between $+1$ and -1. Then $g(x)$ can be written as

$$g(x)=\sum_{m=0}^{N}(-1)^m \operatorname{rect}\left[\frac{x-\dfrac{(x_{m+1}+x_m)}{2}}{x_{m+1}-x_m}\right], \qquad (3)$$

where

$$\operatorname{rect}[x]=\begin{cases} 1, & \text{if } |x|\le 0.5, \\ 0, & \text{if } |x|>0.5, \end{cases} \qquad (4)$$

and $x_0=0$, $x_{N+1}=0.5$. Because the grating is periodic in the x-direction, we may write $g(x)$ in the form of the Fourier series:

$$g(x)=\sum_{n=-\infty}^{\infty} G(n)\exp(i2\pi nx), \qquad (5)$$

where the Fourier coefficients $G(n)$ which give the amplitudes of the diffraction orders in the

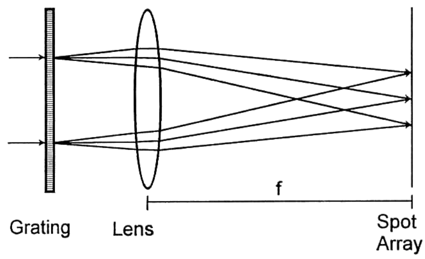

Fig. 1 Setup of spot array generation using a Dammann grating.

output plane are given by

$$G(n)=\int_{-1/2}^{+1/2} g(x)\cos(2n\pi x)\mathrm{d}x \quad (6)$$

where

$$G(0)=2\sum_{m=0}^{N}(-1)^m(x_{m+1}-x_m),$$
$$=4\sum_{m=0}^{N}(-1)^{m+1}x_m+(-1)^N, \quad (7\text{-}1)$$

$$G(n)=\frac{1}{N\pi}\sum_{m=0}^{N}(-1)^m[\sin(2\pi nx_{m+1})-\sin(2\pi nx_m)],$$
$$=\frac{2}{N\pi}\sum_{m=0}^{N}(-1)^{m+1}\sin(2n\pi x_m). \quad (7\text{-}2)$$

Since $G(n)$ are symmetric and real valued, i. e., $G(n)=G(-n)$ and $G^*(n)=G(n)$, the complexity of optimization can be reduced. Moreover $G(n)$ are normalized in such a way that following relation holds.

$$\sum_{n=-\infty}^{\infty}|G(n)|^2=1. \quad (8)$$

3. Optimization of Binary Phase Grating

3.1 Conventional spot array

The objective here is to determine some binary grating structures which can give equally intense diffraction orders in the range of $-N$th to Nth, together with each diffraction intensity being as high as possible, i. e.,

$$I(0)=I(\pm1)=I(\pm2)=\cdots=I(\pm N). \quad (9)$$

We need N transition points in a half-period to control $2N+1$ central diffraction orders and there exist 2^{N-1} different solutions[2] for the Dammann grating. This is usually related to the nonlinear optimization problem. Various optimization techniques have been used to solve this problem including simulated annealing and the Newton-Raphson method. Here, we employ the gradient descent method. The merit function E is defined as the sum of squared errors between the target and the actual diffracted intensity as follows:

$$E=\sum_{n=0}^{N}[t(n)-I(n)]^2, \quad (10)$$

where $t(n)$ is a target value of nth order and $I(n)$ is the actual intensity of the nth diffraction order and is defined as

$$I(n)=|G(n)|^2. \quad (11)$$

There may be various target values $t(n)$. A usual choice of $t(n)$ for conventional spot array is the mean value of the diffracted intensities:

$$t(n)=\frac{1}{2N+1}\sum_{m=-N}^{N}I(m). \quad (12)$$

In an iterative manner, a new set of transition points $\{x'_i\}$ is determined from the previous set $\{x_i\}$ by gradient descent of the merit function E as follows:

$$\Delta x_i=-\beta\sum_{m=1}^{N}\frac{\partial E}{\partial x_m},\quad \text{for } i=1,2,\ldots,N \quad (13)$$

where $\Delta x_i=(x'_i-x_i)$, and the adapting rate β is a constant that governs the stability and rate of convergence. Equation (13) can be written as the product of two parts by using the chain rule:

$$\frac{\partial E}{\partial x_m}=\sum_{n=0}^{N}\left[\frac{\partial E}{\partial I(n)}\cdot\frac{\partial I(n)}{\partial x_m}\right]. \quad (14)$$

The first component on the right-hand side of Eq. (14) is the derivative of the merit function E with respect to the intensity $I(n)$, and the second component is the derivative of the intensity with respect to the transition points x_m as follows:

$$\frac{\partial E}{\partial I(n)}=-2[t(n)-I(n)], \quad (15)$$

$$\frac{\partial I(0)}{\partial x_m}=2G(0)\frac{\partial G(0)}{\partial x_m},$$
$$=(-1)^{m-1}8\left[4\sum_{s=1}^{N}(-1)^{s-1}x_s+(-1)^N\right], \quad (16)$$

$$\frac{\partial I(n)}{\partial x_m}=(-1)^{m+1}\frac{16}{n\pi}\cos(2n\pi x_m)$$
$$\cdot\sum_{s=1}^{N}(-1)^{s+1}\sin(2n\pi x_s),\quad n\neq0. \quad (17)$$

3.2 Selective-order missing spot array

The configuration of selective-order missing (SOM) spot array is obtained by canceling out unwanted diffraction orders among the conventional spot arrays. The algorithm of SOM spot array design is basically the same as that of the conventional spot array except for the target value $t(n)$ of Eq. (12). As a special example of SOM, we consider the BPG structure of the zeroth-order missing (ZOM) spot array. It is designed not to have zeroth order in the range of $-N$th to Nth diffraction orders. The target value $t(n)$ of Eq. (12) should then be modified as follows:

$$t(0)=0,$$
$$t(n)=\frac{1}{2N}\sum_{m=-N}^{N}I(m),\quad n\neq0. \quad (18)$$

For the ZOM spot array we can derive a constraint which makes $G(0)$ zero from Eq. (7-1) :

$$4\sum_{m=1}^{N}(-1)^{m-1}x_m+(-1)^N=0 \quad \text{or}$$

$$x_N=\sum_{m=1}^{N-1}(-1)^{N+m-1}x_m+\frac{1}{4}. \qquad (19)$$

For $N=3$, for example, there are 4 ($=2^{N-1}$) different solutions of 1×7 conventional spot array. However, in using Eq. (19), the number of independent transition points reduces to 2 in the 1×6 ZOM spot array. We may easily obtain the two solutions for the 1×6 ZOM spot array, but the best one is $x_0=0.0000$, $x_1=0.0565$, $x_2=0.2977$, $x_3=0.4913$, $x_4=0.5000$. It is noted that the solution satisfies the constraint of Eq. (19), that is, $x_3=x_2-x_1+0.25$. The efficiency is about 74.5%, so the relative intensities of the seven beams are about $I_0=0.0$ and $I_i=0.124$ (±1, ±2, and ±3), as shown in **Fig. 2.**

Another structure of the SOM spot array is the even-order missing (EOM) spot array. In the EOM spot array, all even-order diffracted beams should be suppressed as follows :

$$I(\pm1)=I(\pm3)=\ldots=I(\pm N),$$
$$I(0)=I(\pm2)=\ldots=I(\pm(N-1))=0. \qquad (20)$$

Therefore, the target value $t(k)$ should be modified in the EOM spot array as follows :

$$t(n)=\frac{1}{N+1}\sum_{m=-N}^{N}I(m), \quad n,\ m \text{ are odd},$$
$$t(n)=0, \qquad n \text{ is even}. \qquad (21)$$

A useful constraint of EOM spot array for the symmetric binary phase gratings can also be derived from Eq. (6). It is noted that if n is odd, then $\cos(2n\pi x)$ in Eq. (6) is an odd function with respect to the axis $x=1/4$ (center of the half-period), whereas, if n is even, then it is an even function with respect to the axis $x=1/4$. We can make the phase transmission function $g(x)$ as an odd function with respect to the axis $x=1/4$ as follows :

$$g(x)=-g(1/2-x), \quad 1/4<x<1/2. \qquad (22)$$

Substitution of Eq. (22) into Eq. (6) leads to the result that all the even orders have zero amplitude as follows :

$$G(n)=0, \quad n \text{ is even},$$
$$G(n)=4\int_0^{1/4}g(x)\cos(2n\pi x)\mathrm{d}x, \quad n \text{ is odd}. \qquad (23)$$

The equivalent expressions of Eq. (22), in terms of transition points x_i, are given by

$$x_{(N+1)/2}=1/4,$$
$$x_i+x_{N+1-i}=1/2, \quad i=1,2,\ldots,(N-1)/2. \qquad (24)$$

Since the degree of freedom (complexity) for designing the EOM grating is decreased from N to $(N-1)/2$ due to the constraints of Eq. (24), it is sufficient to obtain the information of the grating structure of only one quarter of the full period. Here is an example of getting a solution for 1×6 spot arrays. Since the initial configuration of the BPG structure is the same as that of the 1×11 spot array, there exist 5 transition points between $x_0=0.0$ and $x_6=0.5$. From Eq. (24), we can obtain the conditions for the transition points of the EOM spot array: $x_3=0.25$, $x_4=0.5-x_2$, and $x_5=0.5-x_1$. There remain only two independent variables x_1 and x_2, which are determined by the optimization rule of Eqs. (13) and (17). A

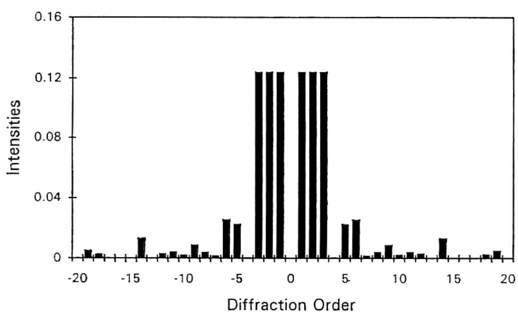

Fig. 2 Diffraction intensities of the 1×6 zeroth-order missing spot array. Efficiency is about 74.5%.

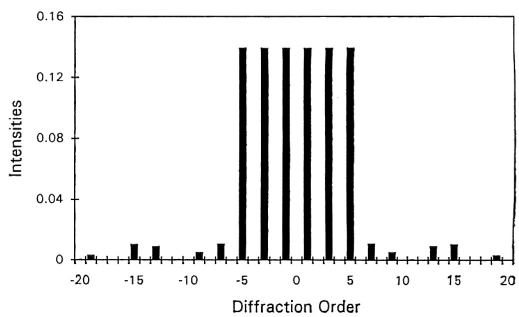

Fig. 3 Diffraction intensities of the 1×6 even-order missing spot array. Efficiency is about 83.6%.

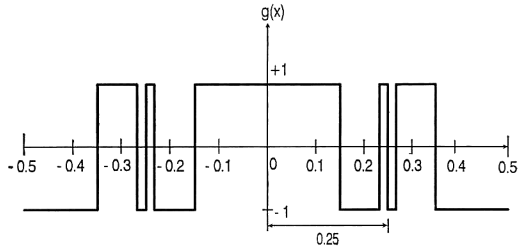

Fig. 4 One period of a binary phase transmission function of the even-order missing structure.

set of transition points is given by [0.0, 0.145516, 0.242941, 0.25, 0.257059, 0.354484, 0.5]. The diffraction efficiency for the central 6 spots is 83.57%, and the relative intensities of the diffraction orders are $I_i=0.0$ (i=even), and $I_j=0.139$ ($j=\pm1, \pm3$, and ±5) as shown in **Fig. 3. Figure 4** shows the single period of the EOM design producing the 1×6 spot array. One may see that the structure is symmetric with respect to the midpoint of a period and antisymmetric with respect to the center of the half-period as expected in Eqs. (22) and (24). It has been reported that the even-numbered spot arrays can also be produced by the translation of half of the period into the second half upon adding a π phase shift. It is interesting to note that our results of EOM spot array agree with the translation and reflection symmetry reported by Morrison.[12]

4. Results and Discussion

The solutions up to 1×20 EOM spot array gratings are given in the **Table 1.** Transition points x_n, efficiency η, and minimum distances $\delta_{\min}$ between the nearest neighbor of transition points in the grating structure are presented. Though there exist $2^{(M/2-1)}$ possible solutions for the $1\times M$ spot array, the most efficient one is selected for each M. The results show that the efficiencies are in the range of 63%-84%.

The speed of convergence of the optimization depends on the adapting rate β and the transition point N. **Figure 5** shows the mean square errors versus iteration number for several values of β. As can be seen, when β is small, it takes a long time to reach a minimum value. As β increases, the optimization time decreases, but it tends to oscillate around the minimum.

Symmetries obtained in Sec. 3 reduce the number of independent parameters as follows:

$$k=\begin{cases} N-1, & \text{for ZOM } (N=1,2,3,\dots) \\ (N-1)/2, & \text{for EOM } (N=1,3,5,\dots) \end{cases} \qquad (25)$$

where N is the number of transition points in half-period, and k is the number of independent parameters. The diffraction efficiency depends on the configuration of signs of the diffraction amplitude $G(n)$. Theoretical considerations predict that the number of solutions having different efficiencies at each k value is 2^k for ZOM and EOM spot arrays. For a small value of k (up to $N=7$), the complete solutions have been found empirically. There are, for example, 7 transition points in a half-period of the grating for the 1×8 EOM spot array. The number of independent transition point is 3 from Eq. (25). We have obtained 8 solutions

Table 1 Solutions of even-order missing spot array gratings. Transition points x_n, efficiencies η, and minimum distance $\delta_{\min}$ are listed.

$1\times M$	x_n	η	$\delta_{\min}$
2	0.0000, 0.2500, 0.5000	81.06	0.2500
4	0.0000, 0.0272, 0.2500, 0.4728, 0.5000	70.64	0.0544
6	0.0000, 0.1455, 0.2429, 0.2500, 0.2571, 0.3545, 0.5000	83.57	0.0071
8	0.0000, 0.0658, 0.0817, 0.1533, 0.2500, 0.3467, 0.4183, 0.4342, 0.5000	69.76	0.0159
10	0.0000, 0.0444, 0.0930, 0.1935, 0.2005, 0.2500, 0.2995, 0.3066, 0.4070, 0.4556, 0.5000	71.03	0.0071
12	0.0000, 0.0013, 0.0972, 0.1621, 0.2104, 0.2325, 0.2500, 0.2675, 0.2896, 0.3379, 0.4028, 0.4987, 0.5000	69.66	0.0026
14	0.0000, 0.0031, 0.0895, 0.1068, 0.1154, 0.1650, 0.2164, 0.2500, 0.2836, 0.3350, 0.3847, 0.3932, 0.4105, 0.4969, 0.5000	67.38	0.0061
16	0.0000, 0.0014, 0.0410, 0.0873, 0.1349, 0.1480, 0.2153, 0.2303, 0.2500, 0.2697, 0.2847, 0.3520, 0.3651, 0.4127, 0.4590, 0.4986, 0.5000	63.33	0.0029
18	0.0000, 0.0018, 0.0263, 0.0670, 0.0907, 0.1690, 0.1731, 0.1927, 0.2308, 0.2500, 0.2692, 0.3073, 0.3269, 0.3310, 0.4093, 0.4330, 0.4737, 0.4982, 0.5000	62.66	0.0037
20	0.0000, 0.0180, 0.0523, 0.0630, 0.0945, 0.1219, 0.1288, 0.1367, 0.1878, 0.1969, 0.2500, 0.3031, 0.3122, 0.3633, 0.3712, 0.3781, 0.4055, 0.4390, 0.4472, 0.4820, 0.5000	63.29	0.0069

with different efficiencies of 69.8%, 63.8%, 56.7%, 56.6%, 51.5%, 51.3%, 47.5%, and 34.0%.

Figure 6 represents the time per iteration versus the number of independent transition points k. It fits well with the following empirical equation :

$$f(k)=ak^4+bk. \quad (26)$$

where $f(k)$ is the function of computation time per iteration for a given k. The coefficients a and b are constant depending on the computer system. When we use an IBM PC compatible with Intel 80486 CPU, the values of a and b were found to be 3.64×10^{-5} and 9.04×10^{-3}, respectively. It takes 30 ms per iteration for 1×8 ($k=3$) spot array design, therefore 6 seconds is required when total iteration is 200. For 1×20 ($k=9$) spot array it takes about one minute for the same iteration. However, it will take about 12 hours to obtain a 1×100 spot array design.

Besides EOM and ZOM spot arrays, there are different structures of SOM spot array which can be obtained depending on the case. An example is a structure which gives out a 1×8 irregular spot array with selectively missing 0th and $\pm$3th orders. The solution set of the structure is given by [0.0, 0.059028, 0.134828, 0.306527, 0.386516, 0.406412, 0.5]. The theoretical intensity profile for the central diffraction efficiency of 66.97% is represented in **Fig. 7. Figure** 8(a) is the structure of single period of an 8×8 even-order missing binary phase grating. The designed BPGs are also fabricated using the conventional e-beam lithographic techniques on a quartz substrate. Figure 8(b) shows a spot array generated by the grating of Fig. 8(a), which was measured by a charge coupled device (CCD) video camera.

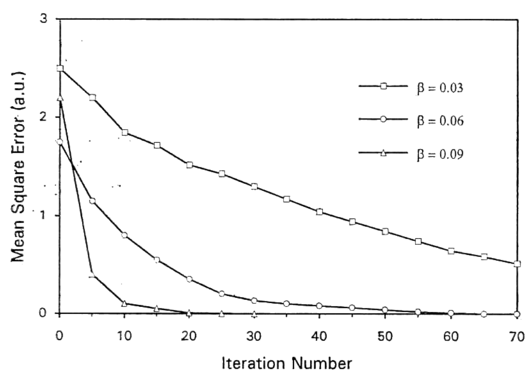

Fig. 5 Change of the mean square error with iteration number.

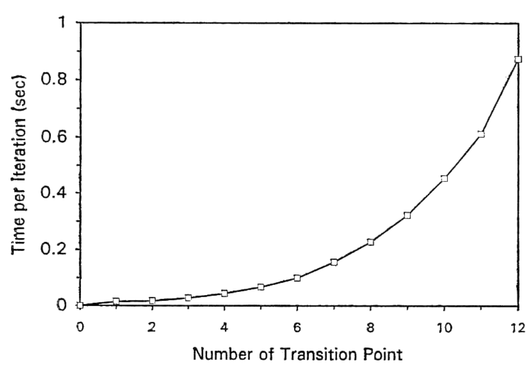

Fig. 6 Time per iteration as a function of the number of independent transition points.

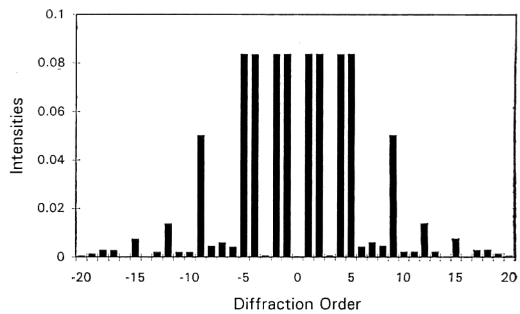

Fig. 7 Diffraction intensities of the 1×8 irregularly spaced selective-order missing spot array with efficiency 66.97%. Missing orders are -3, 0, and $+3$.

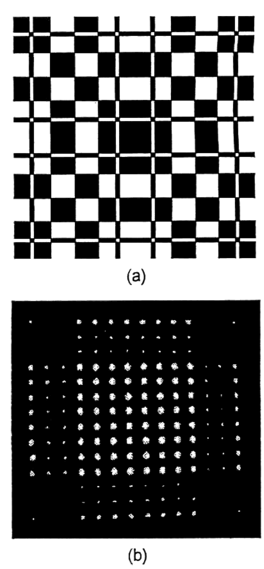

Fig. 8 A period of the grating for 8×8 even-order missing spot array (a), and *its experimental* result (b).

4. Conclusions

We have developed an algorithm for designing the binary phase gratings of selective-order missing spot arrays which selectively cancel or suppress unwanted diffraction orders. Principles and various solutions of the zeroth-order missing and the even-order missing spot arrays are presented. The parameters which control the speed of the design and the constraints which reduce the number of the independent variables are discussed. As a preliminary result of the fabrication of the designed grating, the even-order missing 8×8 spot array is also demonstrated.

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